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Noise Reduction in CT Images in the Domain of the Hyperbolic Wavelet Transform

Miroslav PETROV

Faculty of Mathematics and Informatics, St. Cyril and St. Methodius University of Veliko Tarnovo, Veliko Tarnovo, Bulgaria

ORCID: 0000-0002-5137-6812

m.petrov@ts.uni-vt.bg

Abstract. The quantum and electronic noise generated by the physical principles of X-ray Computed Tomography (CT) degrade the quality of the projection data obtained. These components, as well as the possible anatomical noise, are propagated as pixel noise in the obtained CT images by means of reconstruction algorithms. The different characteristics of the medium through which the X-rays pass determine the direction and the non-stationary nature of the noise in the CT slices. This determines our choice of an adaptive statistical noise reduction algorithm based on the wavelet-threshold method and the correlation between two quasi-identical images. The hyperbolic wavelet transform (HWT) is used in the present paper, because the Hilbert transform (HT) does not change the variance of a random variable. The conducted experiments show that the proposed method gives results comparable or superior to the corresponding ones obtained by similar methods.

Keywords: Entropy of Shannon, Hyperbolic Wavelet Transform, Statistical Noise Reduction, X-ray Computed Tomography.

1. Introduction

During X-ray computed tomography, tissues and structures inside the body are scanned, and their visualization is based on their ability to absorb X-ray photons. The raw data (sinogram) obtained by the detectors when projecting a particular part of the body at different angles sets the image of the scanned object during the Radon transformation. The CT slice is obtained by the inverse Radon transformation applied to the sinogram.

The noise in the CT image is divided mainly into quantum and electronic. The first one is due to the photons measured by detectors, and the second - to the equipment used. Through projection data reconstruction algorithms, it is transformed into pixel noise in the CT slice. Since the decrease in the intensity of the penetrating X-rays depends on the amount of the substance and its density, the noise in the CT image is non-stationary and directed, characterizing the direction of the strongest attenuation.

The CT scan protocol implies a certain compromise between the noise level and the radiation dose. Lower radiation doses decrease the signal-to-noise ratio and the information content of the image, which reduces its diagnostic value. Conversely, the noise level decreases when increasing the radiation dose, but this leads to risks to the patient's health.

Therefore, the development of more sophisticated detectors and new methods for image processing are topical tasks (Gruber et al., 2011; Liu et al., 2011). The quality of such algorithms requires a reduction in the noise level in the resulting slices, as well as improved resolution characteristics without increasing the radiation dose. Various noise suppression techniques have been proposed in the CT scanned images. In terms of implementation, they can be defined as follows: methods in the sinogram space, methods in the field of the reconstructed images, and iterative-reconstructive algorithms (Ehman et al., 2014). The present paper will not discuss the conditions for their applications or their advantages and disadvantages. Detailed information on some methods of noise reduction in CT images is presented in (Kaur et al., 2018; Diwakar and Kumar, 2018).

In the field of multiscale transformations, the most commonly used techniques for reducing noise in images are the wavelet shrinkage methods. The evaluated image is obtained by the inverse conversion of the wavelet coefficients obtained after the corresponding threshold processing. There are various ways to determine the threshold values: Visu Shrink; Sure Shrink; Bayes Shrink, etc. (Donoho and Johnstone, 1994).

Noise reduction algorithms using certain statistical distribution models may not lead to the desired results due to inaccurate representation of the actual noise characteristics in the CT image. Using the images reconstructed from the even and odd data in the sinogram, the paper presents a locally adaptive algorithm for noise reduction in CT slices by means of threshold processing in the field of HWT. The next section is devoted to the motivation and theoretical foundations of the method, and the multiscale HWT is described in Section 2.3. Section 3 contains a detailed description of the method in question, and Section 4 presents the results of the conducted experimental studies and the corresponding comparative evaluations. In the concluding part, there are some notes on the method, the results obtained and the corresponding conclusions.

2. Motivation and Theoretical Foundations

2.1. Motivation and Reasoning

In (Tischenko et al., 2005b), the authors investigate the effect of the patient’s inevitable movements during exposure on the X-ray image in different radiographs. Significant changes of small indistinguishable anatomical structures are observed resulting in the so-called anatomical noise. Comparing two X-ray images of the same anatomy obtained with small changes in the geometry of the image, they propose a method for reducing the anatomical noise. The algorithm is implemented in the wavelet domain, taking into account the correlation between the corresponding characteristics of the two images. When there are no geometric changes, the reduction refers to the quantum and electronic noise.

Using the fact that, unlike structural information, the noise in two quasi-identical projection X-ray images of the same object is almost uncorrelated in time, the authors suggest a noise reduction method (Tischenko et al., 2005a). This wavelet shrinkage method makes use of an appropriate similarity criterion for both images in order to obtain the corresponding weighting matrices. Thus, the weighting wavelet coefficients are set, and, by means of inverse transform, the evaluated image is obtained based on them.

2.2. The Wavelet Shrinkage Denoising Method

The Adaptive Wavelet Shrinkage (AWShrink) method for noise reduction in CT images announced in (Borsdorf et al., 2008) has been considered in connection with the method proposed in this paper. It is adjusted to the noise by statistically evaluating its locally and orientationally dependent strength.

The proposed method for noise reduction in CT slices uses the results of (Tischenko et al., 2005a, b) and can be combined with various methods for projection data reconstruction. It is based on the assumption that the sinogram consists of structural information and time-uncorrelated noise. The required pair of quasi-identical images can be obtained in different ways: reconstructing two disjoint sinogram subsets; Dual source CT scanners; two consecutive scans without geometric changes in the object.

The main steps of the AWShrink method are: obtaining the pair of structurally identical input CT slices; wavelet-decomposition of input images; noise assessment by their high-frequency coefficients; averaging and threshold processing; obtaining the noise-estimated image by inverse wavelet transform.

2.3. The Hyperbolic Wavelet Transform

In (Klih et al., 2004), an approach is proposed for constructing a multiscale transformation with a hyperbolic wavelet based on the theory of generalized functions (distributions). The wavelet transform uses the hyperbole family $(\alpha\pi x)^{-1}$ as basic functions, where α is the scale factor. To determine the values of the distribution at the point of discontinuity $x=0$, the function $G_0(x)=\theta(x-\varepsilon)-\theta(x-\gamma)$ is used, where $\theta(x)$ is the Heaviside function and $0<\varepsilon<\gamma$.

To improve the noise immunity of the hyperbolic wavelet transform, a modified hyperbolic wavelet is used in the contour segmentation problem (Krylov et al., 2006). For this purpose, the adaptive function $G_0(x)$ is replaced by $G(x)=\begin{cases} G_0(x), & |x|>\varepsilon \\ x\cdot\varepsilon^{-1}, & |x|\le\varepsilon \end{cases}$. The modified hyperbolic transformation filters are obtained by the discretization of the function $\psi\left(\frac{x}{s}\right)=\frac{1}{\pi\alpha x\sqrt{s}}G\left(\frac{x}{s}\right)$. In the present paper, the discrete hyperbolic wavelet transform is realized by the convolution of each of the rows and columns of the image with filters of the following type: $g_s=\left\{-\frac{1}{10},\ldots,-\frac{1}{1+2s^{-1}},-\frac{1}{1+s^{-1}},-1,\right.$ $\left.,-1+s^{-1},\ldots,-s^{-1},0,s^{-1},\ldots,1-s^{-1},1,\frac{1}{1+s^{-1}},\frac{1}{1+2s^{-1}},\ldots,\frac{1}{10}\right\}$.

3. The Proposed Denoising Method

The proposed technique for noise reduction in CT images contains three main stages: obtaining the pair of input images; statistical assessment of the noise and determining the adaptive threshold constant; obtaining the denoised image. The sequence of the individual steps of the proposed noise reduction algorithm is shown in Figure 1.

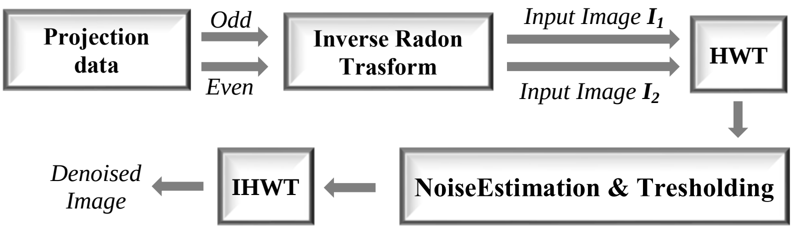


Fig. 1. Methodology of the proposed noise reduction method.

3.1. Obtaining the Input Images

There are various possibilities for obtaining a pair of quasi-identical input CT images. This can be done with a single-source CT scanner or a dual-source scanner. With a single-source CT scanner, it is necessary to scan the object twice in succession, under identical conditions. In order to avoid exposing the patient to radiation twice, these images can be obtained in a single scan, by two separate reconstructions of two non-intersecting subsets of the complete set of projection data. In addition, the averaged image of the two reconstructions corresponds to the image reconstructed from the full set of projections (Natterer, 1986). An example of such subsets are respectively the sets of odd and even projections obtained at a single scan.

3.2. Determining the Adaptive Threshold Constant

Let I_1 and I_2 be the input images obtained by a separate reconstruction of the even and odd numbered projections, assuming that the total number of the projection data is an even number. The way of dividing the projection data provides the relation between the respective standard deviations of the noise in $m^{\text{-}th}$ pixel: $\sigma_m(I_1) \approx \sigma_m(I_2)$. Furthermore, the noise level in each of the images is increased by a factor of $\sqrt{2}$ compared to the noise level in the reconstructed slice of the full set of projections (Natterer, 1986). It follows from the above that the standard deviation of the noise in the input images, and therefore in their averaged image $I = \frac{I_1 + I_2}{2}$, can be determined by the noise image $D = I_1 - I_2$. As a consequence of the fact that the noise in separately reconstructed slices is uncorrelated, the zero covariance between the two input images is also obtained (Borsdorf, 2009). Then, $\sigma(I_1) = \sigma(I_2) = \frac{\sigma(D)}{\sqrt{2}}$, and

$\sigma(I)=\frac{\sigma(I_1)}{\sqrt{2}}=\frac{\sigma(D)}{2}$.

Let $w_{s,m}(\bullet)$ be the coefficient of the m-th pixel, at a scale of s, obtained through HWT. Since the transformation used is linear, the following equality is true: $w_{s,m}(D)=w_{s,m}(I_1)-w_{s,m}(I_2)$. Because the noise in CT images is non-stationary, a local assessment of its standard deviation is required. For this purpose, for each pixel m, a square neighbourhood Ω_m, containing n pixels is examined and, then, $\sigma_{s,m}(D)=\left(n^{-1}\sum_{\bar{m}\in\Omega_m} w_{s,\bar{m}}^2(D)\right)^{1/2}$, since the average noise value is zero. Then, the local values of the standard deviation of the noise in the averaged image are obtained from the equality $\sigma_{s,m}(I)=\frac{\sigma_{s,m}(D)}{2}$. The threshold value adapted to the wavelet parameters is determined by the expression $\tau_{s,m}(r)=\frac{r\cdot\sigma_{s,m}(D)}{2}$, where the parameter r regulates the amount of suppressed noise. The optimal value r_0 of this parameter, at the scale s, is determined on the basis of the relative change in the entropy of Shannon (see Petrov, 2021, Sect. 3.3).

3.3. Obtaining the Noise-Estimated Image

There are different rules for threshold processing of the wavelet coefficients in shrinkage methods, with noise reduction in the obtained data. The most popular ones are the nonlinear functions of the “hard” and “soft” thresholds introduced by Donoho and Johnstone (1994).

The main idea of the method is to preserve the wavelet coefficients, which carry the structural information of the image, and to zero the insignificant coefficients. Due to the discontinuity of the function, small changes in the processed data become a problem for hard threshold processing. The proposed algorithm employs the continuous function of the soft threshold. The estimated wavelet coefficients of image I are $w_{s,m}^0(I)=\operatorname{sgn}(w_{s,m}(I))\cdot\max\left(\left|w_{s,m}(I)\right|-\tau_{s,m}(r_0),0\right)$, by means of which the estimated original CT image is obtained through Inverse HWT (IHWT).

4. Experimental Results and Comparative Analysis

Two types of measures are used to evaluate the quality of the denoised CT images. They are based on:

- Pixel Difference Measurement – Mean Absolute Error (MAE) and Peak Signal-to-Noise Ratio (PSNR);
- Human Visual Measurement – Structural Similarity Index (SSIM) and Universal Image Quality Index (UIQI).

The algorithms for calculating the listed measures are implemented in Matlab. Two experiments are conducted in order to evaluate the effectiveness of the proposed method for reducing noise in CT images.

In Section 2, it was noted that the proposed method for estimating and reducing noise in CT slices was generated by the research published in (Borsdorf et al., 2008; Tischenko et al., 2005a,b). The main objectives of the planned experimental studies and the corresponding comparative analysis are determined based on this basis. The first goal is to evaluate the performance of some multiscale transformations in the adaptive wavelet-shrinking methods for reducing pixel noise using a pair of non-intersecting subsets of synogram data. Then, the different approaches for determining the local threshold constants as a function of the noise characteristics are compared.

4.1. Test CT Image

A medical 16-bit monochrome CT-MONO2-16-ankle.dcm with resolution 512×512 is used as a test image in Matlab workspace, to which Gaussian noise has been added. Each of the images obtained is subjected to the Radon transform in order to obtain and separate the projection data, numbered by odd and even numbers respectively. At the next stage, the corresponding reconstructed CT slices are decomposed by HWT, at a pre-selected level of decomposition.

Table 1. MAE, PSNR and SSIM for the CT-MONO2-16-ankle.dcm image, at the respective noise levels

σ [%]	**AWShrink**			**RWT-based method**			**Shearlet-based method**			**Proposed method**		
	MAE	PSNR [dB]	SSIM	MAE	PSNR [dB]	SSIM	MAE	PSNR [dB]	SSIM	MAE	PSNR [dB]	SSIM
10	1.57	30.43	0.70	0.78	39.56	0.83	0.24	37.89	0.95	0.21	40.34	0.96
20	1.7	27.86	0.48	1.33	33.49	0.66	0.32	27.83	0.94	0.27	38.25	0.95
30	1.78	26.44	0.32	1.67	28.89	0.54	0.49	33.68	0.91	0.36	35.43	0.93

Table 1 provides the corresponding values for three of the four of those quality measures obtained when using the four methods: AWShrink; RWT-based method (Petrov, 2019); Shearlet-based method (Petrov, 2021) and the Proposed method. The comparative analysis, which has been conducted, shows that the proposed method

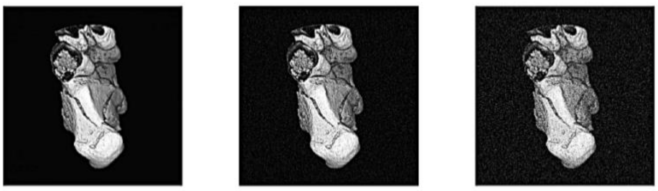
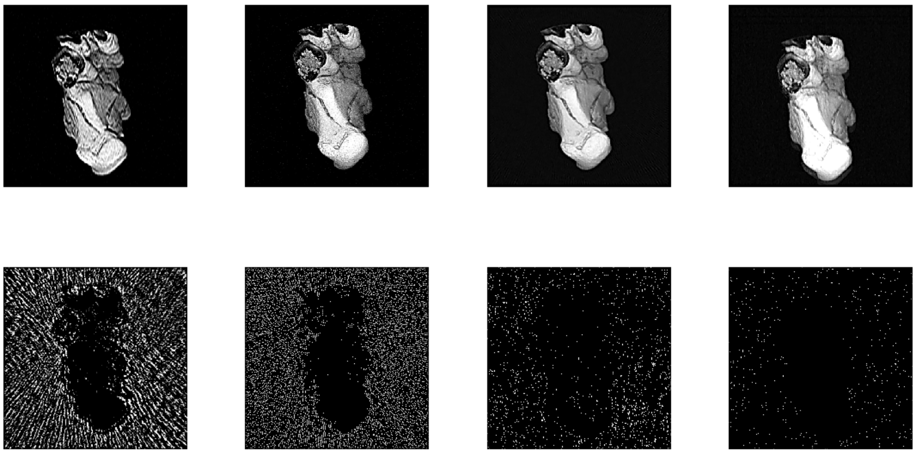
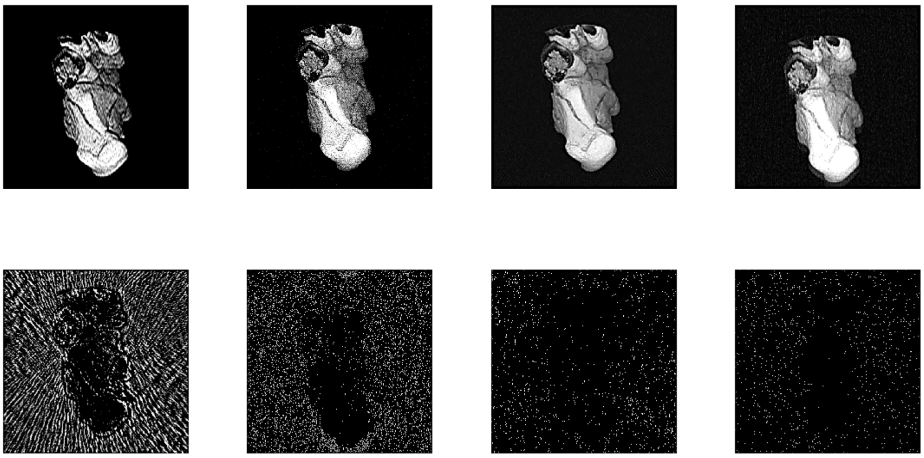

(a) (b) (c)

Fig. 2. Test images: (a) original image; (b) noisy image, $\sigma = 10$; (c) noisy image, $\sigma = 20$.



(a) (b) (c) (d)

Fig. 3. Denoised results of CT-MONO2-16-ankle.dcm (noise level $\sigma = 10$) obtained by the four wavelet-shrinkage methods: (a) Denoised image and the corresponding residual information, obtained through AWShrink; (b) Denoised image and the corresponding residual information, obtained through RWT-based method; (c) Denoised image and the corresponding residual information, obtained through shearlet-based method; (d) Denoised image and the corresponding residual information, obtained through the proposed method.



(a) (b) (c) (d)

Fig. 4. Denoised results of CT-MONO2-16-ankle.dcm (noise level $\sigma = 20$) obtained by the four wavelet-shrinkage methods: (a) Denoised image and the corresponding residual information, obtained through AWShrink; (b) Denoised image and the corresponding residual information, obtained through RWT-based method; (c) Denoised image and the corresponding residual information, obtained through shearlet-based method; (d) Denoised image and the corresponding residual information, obtained through the proposed method.

achieves higher values for the PSNR and SSIM, as well as lower values for the MAE, in comparison with the other methods considered for all noise levels.

Figure 2 shows the original image and the images to which Gaussian noise has been added with a standard deviation of $\sigma = 10$ and $\sigma = 20$ respectively.

Figures 3 and 4 show the results obtained when using some multiscale tranforms noise reduction methods, whose standard deviation is $\sigma = 10$ and $\sigma = 20$ respectively.

4.2. Real CT Image

The purpose of the next experiment is to evaluate visually the effectiveness of the proposed method, as well as to confirm it by means of the quantitative measure UIQI. The comparative analysis is again conducted by means of the multiscale methods in 4.1, using real CT image of a pancreas as shown in Figure 5. This image was obtained from publicly available medical databases (see WEB) and are in the DICOM format.

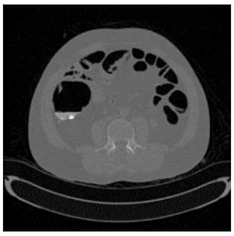
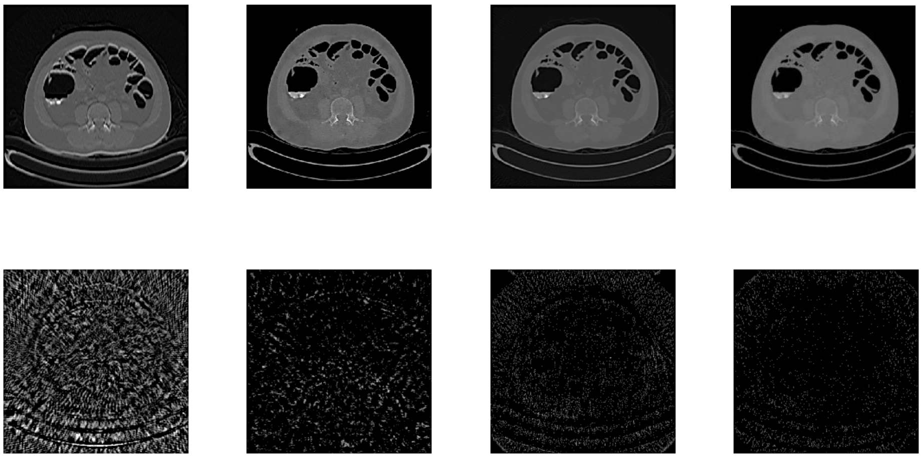


Fig. 5. Real CT image.



(a) (b) (c) (d)

Fig. 6. (a) Denoised image $(UIQI = 0.632)$ and the corresponding residual information, obtained through AWShrink; (b) Denoised image and the corresponding residual information, obtained through RWT-based method $(UIQI = 0.918)$; (c) Denoised image and the corresponding residual information, obtained through shearlet-based method $(UIQI = 0.948)$; (d) Denoised image and the corresponding residual information, obtained through the proposed method $(UIQI = 0.973)$.

Figure 6 presents the denoised CT image and the corresponding images containing the residual information, which has been removed from the image when applying the methods in question. In addition, the corresponding values obtained for UIQI are presented.

5. Concluding Remarks

Phantom and real CT images are used to assess the qualities of the proposed method. The algorithm used allows to suppress the noise while preserving the structural information and it does not increase the patient's radiation exposure. The proposed methodology is based on the lack of time correlation between the noise components of the pair of quasi-identical CT images obtained from the projection data from a single scan. The HWT filed is chosen to estimate the noise components, because the HT does not change the variance of a random variable. The threshold constant in the proposed wavelet-shrinkage method is determined based on the analysis of Shannon's entropy. The denoised image is obtained by IHWT applied to the averaged and WT-processed coefficients. The conducted tests show that the proposed method achieves higher values of PSNR, SSIM and UIQI, as well as lower MAE values than those resulting from the methods under consideration.

References

Borsdorf, A. (2009). *Adaptive Filtering for Noise Reduction in X-ray Computed Tomography*, PhD thesis, University of Erlangen-Nurenberg, Bavaria, Germany.

Borsdorf, A., Raupach, R., Hornegger, J. (2008). Multiple CT-reconstructions for locally adaptive anisotropic wavelet denoising, *Int. J. Comput. Assist. Radiol. Surg.* **2** (5), 255–264.

Diwakar, M., Kumar, M. (2018). A review on CT image Noise and its denoising, *Biomedical Signal Processing and Control* **42**, 73-88.

Donoho, D. L., Johnstone, I. M. (1994). Ideal spatial adaptation by wavelet shrinkage, *Biometrika* **81** (3), 425-455.

Ehman, E. C.,Yu, L., Mandusa, A., Hara, A.K., Shiung, M. M., Jordal, D., Lake, D. S., Paden, R. S., Blezek, D. J., Bruesewitz, M. R., McCollough, C. H., Hough, D. M., Fletcher, J. G. (2014). Methods for clinical evaluation of noise reduction techniques in abdominopelvic CT, *Radiographics* **34** (4), 849-862.

Gruber. M., Weber. M, Homolka, P., Nemec, S., Fruehwald-Pallamar, J., Uffmann, M. (2011). Feasibility of dose reduction using needle-structured image plates versus powder-structured plates for computed radiography of the knee, *American J. Roentgenology* **197**, 318-323.

Kaur, R., Juneja, M., Mandal, A. K. (2018). A comprehensive review of denoising techniques for abdominal CT images, *Multimedia Tools Applications* **77**, 22735-22770.

Klih, Y. A., Antoshchuk, S. G., Nikolenko, A. A. (2004). Adaptive Basis Function of Hyperbolic Wavelet Transform (Russian), *Proc. of Odessa Polytechnic University* **2** (22), 121-125.

Krylov, V. N., Polyakova, M. V., Volkova, N. P. (2006). Contour gegmentation in spase of hyperbolic wavelet transform with the use of mathematical morphology (Russian), Automatica. *Automation. Electrical Engineering Complexes and Systems* **2** (18), 57-63, available at http://www.aaecs.org.

Liu, X., Shaw, C. C., Lai, C. J., Wang, T. (2011). Comparison of scatter rejection and low-contrast performance of scan equalization digital radiography (SEDR), slot-scan digital radiography and full-field digital radiography systems for chest phantom imaging, Medical Physics 38, 23-33.

Natterer, F. (1986). The Mathematics of Computerized Tomography, Wiley and B. G. Teubner, Stuttgart.

Petrov, M. (2019). CT image denoising based on locally adaptive tresholding, System Research and Information Technologies 4, 39-48.

Petrov, M. (2021). Shearlet-based adaptive noise reduction in CT Images, Iranian Journal of Electrical and Electronic Engineering 17 (1), 1-8.

Tischenko, O., Hoeschen, C., Buhr, E. (2005a). An artifact-free structure-saving noise reduction using the correlation between two images for threshold determination in the wavelet domain, in Fitzpatrick, J. M., Reinhardt, J. M. (Eds.), Proc. of the SPIE, Medical Imaging 2005: Image Processing (April 2005, San Diego, CA, USA), 5747, pp. 1066-1075, available at https://doi.org/10.1117/12.595863

Tischenko, O., Hoeschen, C., Buhr, E. (2005b). Reduction of anatomical noise in medical X-ray images, Radiation Protection Dosimetry 114 (1-3), 69-74.

WEB. The Cancer Imaging Archive, available at https://www.cancerimagingarchive.net.

Authors' information

M. Petrov is an assistant professor at the Department of Computer Systems and Technologies at the St. Cyril and St. Methodius University of Veliko Turnovo (Republic of Bulgaria). He received BS and MS degrees in Communications and Security Technologies and Systems. He was awarded his PhD degree - "Automated systems for Information Processing and Management" from Technical University – Sofia (Republic of Bulgaria), in 2014. His current research interests include Computer Vision, Image Processing, Communication and Computer Systems.

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