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Optimization of Relocation Processes for Shared E-vehicles

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Abstract. This study describes a system with one-way trips and relocations of e-vehicles between sectors by service personnel according to a dynamically compiled list of service trips. The model includes an algorithm that uses model parameter values to optionalize expected income, depending on the dynamically selected e-vehicle transfer. The implementation of the MIP (Mixed-Integer Programming) type algorithm proposed in the study pays particular attention to its performance, as optimization should be performed dynamically with a few hours' interval. The developed optimization algorithm has been validated for its practical application in Riga, Latvia.

Keywords: Shared e-vehicles, optimization of relocation, mixed-integer programming.

1. Introduction

Today vehicle rentals are widespread throughout the world. Vehicle rental approaches tend to be classified in two large groups (Illgen et al., 2019): (1) *traditional rental*—when customers receive and transfer vehicles after use at specially arranged points of leasing firms and rental will take one or more days—and (2) *vehicle sharing*—when vehicles can be taken for use anywhere, even for a very short period of time, and may be left anywhere at the end of the trip. Vehicle sharing has quickly gained popularity. The growth rate of the service has particularly increased during the COVID-19 pandemic, as it allows urban populations to avoid the need to travel to their destination via public transport.

Vehicle sharing solutions are available for both segments – (1) cars, and (2) scooters/ bicycles. This study exclusively focuses on car sharing since the service models between the two segments differ substantially.

The main challenge facing e-vehicle sharing rental systems is to achieve the optimal (the most profitable) deployment of vehicles in a city. This requires relocating e-vehicles quickly to the most profitable sectors of the city which in turn causes additional costs. For example, the studies of (Vasconcelos et al., 2017) show that technical relocation of vehicles take approximately 14% of the total distance carried out by the vehicles. Optimization algorithms are used to address this problem, which gives users or system holders recommendations on the need to relocate vehicles to achieve a "more cost-effective" deployment and, hence, higher returns.

According to (Boyaci et al., 2015) vehicle sharing systems benefit both users and the society in general. The two main benefits for individual users include reduced personal transport costs and improved mobility. Research has shown that car sharing reduces the average number of kilometres travelled by a vehicle and is likely to reduce traffic congestions (Crane et al., 2012) and CO2 emissions (Shaheen et al., 2013). Ensuring mobility at affordable prices for economically vulnerable groups is another societal benefit of such public transport systems highlighted by the authors.

Compared to traditional vehicle rental systems the development and operation of e-vehicle (automotive) sharing systems face additional technological and practical challenges. For example, the relatively limited autonomy of the currently available electric cars requires the recharging of vehicles multiple times in the case of longer trips. This can only be carried out at specific charging stations. Furthermore, e-vehicle charging time is significantly longer than the refuelling of motor vehicles powered by internal combustion engines. Accrued statistics show that an electric vehicle used within the city area must be charged on average every three days. Because of high costs, the number of charging stations is sparse, and the total charging time can be quite long. Charging time is shorter at fast-charging stations. However, due to higher costs, fast-charging stations are rare. Finally, electricity consumption is significantly affected by driving and environmental barriers (e.g., speed, air temperature), which need to be considered when assessing the actual daily charging level of the vehicle (Brandstätter et al., 2016).

This study offers an e-vehicle sharing model that considers the dynamics of relocating vehicles between different sectors in a city. The proposed model is designed to fully meet the requirements of real systems and differs from all known solutions.

The paper is structured as follows: a theorethical background on vehicle sharing models (Section 2), an original vehicle sharing model proposed by the authors (Section 3), a short discussion on the research findings (Section 4), and conclusions (Section 5).

2. Theoretical Background

This chapter deals with the results of other studies. The ideas described in (Gambella et al., 2018) served as a landmark for the solution developed by the authors.

2.1. Review of Sharing Models

Although scientific literature on e-vehicle sharing is broad, the authors of other works as well as the authors of this study conclude that the scientific literature currently available does not offer a model that, along with parameters such as the number, size and location of charging stations, the size of the car fleet, would also take into account the dynamics of vehicle relocations and system balancing when the reserving of e-vehicles is used. The existing models (de Almeida Correia et al., 2012; Lin et al., 2011) either overview station locations without considering vehicle relocations (Lin et al., 2011), or overview station locations, assuming only a limited subset of stations corresponding to the current demand should be serviced (de Almeida Correia et al., 2012). If vehicle relocations are modeled (de Almeida Correia et al., 2012), vehicle movements and associated costs are only considered at the end of the operating period (usually daily) and, therefore, affect the size of the available fleet (Boyaci et al., 2015).

According to (Brandstätter et al., 2016) which studies an example of a city in Southern California, even 3-6 vehicles can be sufficient to provide 100 trips daily and achieve optimal customer waiting times. Meanwhile, about 18–24 vehicles would be enough to reduce the required number of vehicle relocations. The authors conclude that, in addition to the number of vehicles (per trip), the relocation algorithm and the charging approach used are key factors for the successful use of such a system.

(Boyaci et al., 2015) highlights the importance of the service level, which, in his view, influences the access of potential users to vehicle stations, i.e. (1) the distance between the location of the vehicle and the destination resp., from the point of start and arrival of the car, and (2) the availability of vehicles at stations. On the other hand, the number and size of the stations and the size and availability of the vehicle park at “real time” at the “particular station” are affected by the costs of establishing and operating the car sharing system.

According to the classification of (Illgen et al., 2019) the e-vehicle sharing system analyzed in this paper is:

(1) commercial solution as the aim is to generate maximum income,
(2) station-based - vehicles are deployed in any available parking place and we can assume that the city is divided into areas (so-called stations,
(3) one-directional as the customer is allowed not return the vehicle to the start point of the trip,
(4) with relocations as the service staff moves e-vehicles to potentially more favourable places in the city,
(5) without pre-booked trips as an e-vehicle may be rented by the customer anywhere, at any time without prior e-vehicle reservation.

Increased profits for commercial sharing transport systems can be achieved by increasing the relocation efficiency as well as supplying vehicles to the places in a city where customers will need them with the highest probability.

2.2. Approbation of Algorithms

An optimization algorithm that was matching to the task of this study is given in (Gambella et al., 2018). This is the Mixed-Integer Programming (MIP) model, which maximises profits on the assumption that next day trips, the availability of fixed stations and availability of relocation staff are known.

(Gambella et al., 2018) indicates that the algorithm has been tested for a relatively small number of scheduled trips and vehicles: 14 stations, 20 e-vehicles, 2 relocators, 120 trips booked on the previous day. The optimal timing, according to data provided by the algorithm's authors, has been calculated in approximately 20 minutes.

The model described in (Gambella et al., 2018) has a deficiency that prevents it from being introduced for the purpose of this study: It is designed on the assumption that future trips are known, eg, customers order the vehicles indicating starting and ending stations and the duration of the trip. Originally, the authors examined the possibility of predicting customer trips based on the data history of many previous trips. However, the experiments failed to obtain a sufficiently reliable forecast for future trips, so this idea was rejected. On the other hand, if the trips forecast is not sufficiently precise, the model defined by (Gambella et al., 2018) does not provide a credible relocation plan, i.e., the vehicles will possibly be moved to places where customers will not need them. Although a reliable forecast of all daily trips was not obtained, it was found that up to 20% of trips could be predicted up to one hour and station accuracy. This benefit is further used in the author 's solution and is described more precisely in the next chapter.

Consequently, the (Gambella et al., 2018) algorithm is not used directly in the study, but the authors have developed an original algorithm as a part of an optimization model.

3. Vehicle Deployment Model

This chapter provides a brief description of the model that will be precisely defined for shared e-vehicles in the next chapter.

3.1. Station-based Algorithm

According to (Gambella et al., 2018), the continuous division of the transport sharing service area into the sectors (other studies referred to as stations) does not significantly affect optimization. However, division of territory into sectors must be carried out under several conditions. First, the sectors need to be relatively small to place a verhicle in the area for the client to reach within “reasonable” times (the accumulated real data set shows that customers are ready to spend up to five minutes for reaching a vehicle). Secondly, the driving time between two adjacent sectors must be comparable. Thirdly, within one sector, customers' behaviour must be comparable, i.e., customers make trips from the respective sector uniformly frequently.

Riga city is characterized by the following parameters: 614,618 inhabitants in 2021, an area of 307 km2, it is divided into two parts by the river Daugava with 4 bridges for transport, which has an impact on transport flows. The division into sectors was created by analyzing the historical data on the use of e-vehicle shared in the city of Riga. A cluster analysis was performed and areas with similar e-vehicle usage rates created sectors. As mentioned above, the experience of other cities shows that the division into sectors based on population, area, etc. indicators not successful. Although the division of a city into sectors may have a significant impact on revenue in a particular city, the aim of this study is to analyze the methodology of the e-vehicle service, leaving the analysis of the division of the city into sectors to another study.

In analysing the history of trips and knowing the specific characteristics of the area, Riga was divided into sectors as can be seen in Figure 1.

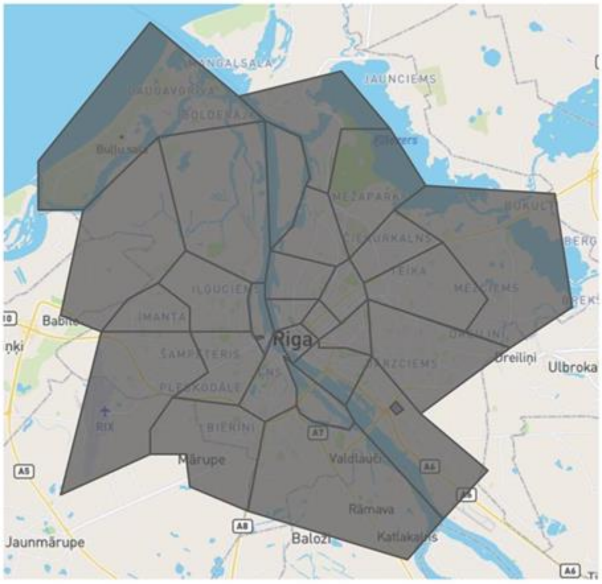

Figure 1. Division of Riga, Latvia in sectors
(using OpenStreetMap: www.openstreetmap.org/copyright)

3.2. Forecasting User Trips

While sharing car users in Riga do not make all requests a day in advance, certain user trips can be scheduled with high possability, using historical trip data. Using cluster analysis, you can find "routine arcs" — trips that consistently start and end the day from day to day in the same sectors, at the same times. The arcs found in this way can be added to the model as A_c (see Chapter 3.4.1).

3.2.1. Cluster Analysis

The routine arc cluster anaylsis can be performed using the DBSCAN (Ester et al., 1996) algorithm available from the Scikit-learn library (Pedregosa. et al., 2011).

The cluster analysis was executed over seven dimensions with varying weights, w, associated with each dimension (i.e., the higher the weight, the more sensitive DBSCAN is to changes in value difference in that dimension):

- latitude of the start location, $w = 0.4\ \mathrm{km}^{-1}$;
- longitude of the start location, $w = 0.4\ \mathrm{km}^{-1}$;

- latitude of the end location, $w = 2.0\ \mathrm{km}^{-1}$;
- longitude of the end location, $w = 2.0\ \mathrm{km}^{-1}$;
- duration of the arc, $w = 30\ \mathrm{min}^{-1}$;
- time of day when the e-vehicle was reserved, $w = 60\ \mathrm{min}^{-1}$;
- customer ID, $w = \infty$ (i.e., all arcs within a cluster must be created by the same customer).

The values for weights were determined empirically with the target to categorize all similar arcs withing the same cluster, yet to avoid merging multiple clusters together. The difference in weights for the start and end locations can be justified as, although a customer cannot precisely choose the starting location (the closest e-vehicle to the customer at the time of reservation can be located over range of possible locations), the end location is purely chosen by the customer. Therefore, the end location tends to be more consistant than the starting location.

The "Manhattan" distance metric was used when performing the cluster analysis. The DBSCAN parameters, ε (neighbourhood size) and minPts (minium required number of points within a neighbourhood), were determined empirically to be:

- $\varepsilon = 1.4$,
- $\mathrm{minPts} = 0.0833 \cdot N$, where N is the number of days in the interval over which the sample of historic data was provided.

An example cluster of routine arcs found using the parameters above and historic data spanning two months can be seen in Figure 2 below. This example shows that the end locations of the arcs tend to be more consistent than the start locations.

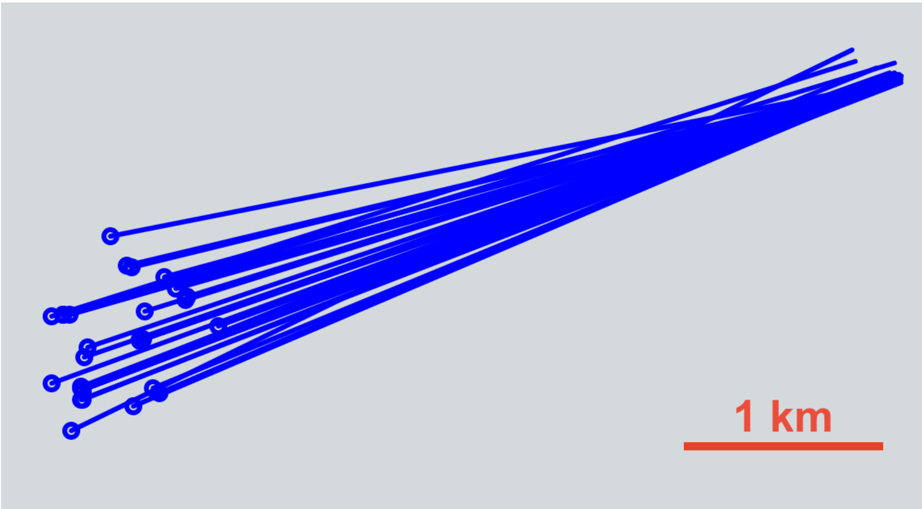

Figure 2. An example cluster of routine arcs. Each line indicates one arc and connects its start (depicted by a circle) and end locations.

When considering the generated clusters, a crucial parameter to take into consideration is the day of week. For example, within some clusters, the arcs were observed to occur every day of the week, whereas other clusters contained arcs that only took place within some particular set of days of the week.

3.2.2. Effectivity of the Cluster Analysis

For example, Figure 3 depicts the percentage of predictable routine arcs over the total number of expected arcs for each hour. This data suggests that within the time interval of 6am to 7am, close to 70% of the expected number of arcs for this time interval can be predicted using the cluster analysis. However, during midday (10am–4pm) only around 10% arcs are routine (i.e., can be predicted using cluster analysis).

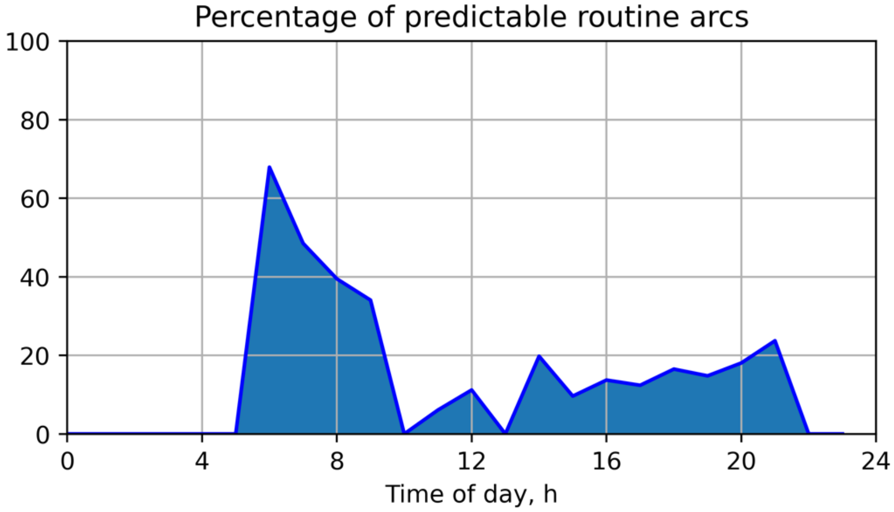

Figure 3. Percentage of predictable routine arcs (with confidence interval of 0.50) over the total predicted number of arcs within an hour interval. This data is for Tuesday.

This data suggests, that although routine arcs can be predicted reasonably well using cluster analysis, during most of the day routine arcs compose only percentage of the total number of arcs. Therefore, other metrics should be used in tandem with predicted routine arcs when planning relocation operations.

3.3. Estimation of Sectors' Income

The purpose of relocation is to place cars in areas where they are in demand and profits are expected accordingly. In sharing car systems with booking in advance, a full estimate of demand and expected profit is known prior to the planning of relocation operations. But in our case, requests are made in real time, without prior bookings of vehicles. Therefore, to take tactical decisions on relocation operations, it is necessary to be able to carry out an alternative assessment to which stations to move the vehicles.

One potential solution that this study looks at is the modelling of expected income using historical data. The model of expected incomes describes the average expected benefits over a specific time period from a vehicle parked in a particular sector that can be rented by users. The expected income most probably will vary from station to station, as well as it will change over a day: In "peak hours" the expected income will be higher, in "quiet hours" less.

Modelling the expected incomes may not only provide tactical support in the planning of resettlement operations, but also give general impression on the behaviour of sharing car users. Comparing the expected income at different times, different weekdays and different stations, it will be possible to draw conclusions that can also help you to make strategic, long-term decisions, such as handling different sectors or deploying charging stations.

Expected income can be described by a function $p(t)$ which changes depending on the time of the clock t (here and on, the clock time is measured in hours from 00:00). The expected income $Y(t_1, t_2)$ for a vehicle that is located in a specific sector between time t_1 and t_2 can by calculated by an integral:

$$Y(t_1, t_2) = \int_{t_1}^{t_2} p(t)\,\mathrm{d}t \tag{1}$$

3.3.1. Estimation of Expected Income Using Historical Data

From the history of car rentals in different sectors, you can do an assessment of expected income in each sector for the next day. H_t denotes a set $h \in H_t$ of all cars located in a specific sector at the time t. $v(h, t, \Delta t)$ denotes the total income from renting a car h in the time interval between t and $t + \Delta t$ (if the car h has not been rented during this period, then the income is $v(h, t, \Delta t) = 0$). This data is taken from historical information about car rentals. The $p(t)$ can be estimated as follows:

$$p(t) = \frac{\sum_{H_t} v(h, t, \mathrm{d}t)}{|H_t| \cdot \mathrm{d}t} \tag{2}$$

Since historical information on the rental of sharing cars is final, it is expected that there will be significant noise in the function $p(t)$ when $\mathrm{d}t$ is small. In order to reduce noise and to increase the robustness of the estimated income $p(t)$, the function $p(t)$ is handled with Gauss Filter $W[p](t)$ (WEB, a) - Weierstrass transform. Since Gauss Filter stores information on total income, it is therefore appropriate for that purpose:

$$\int_0^{24} p(t)\,\mathrm{d}t = \int_0^{24} W[p](\mathrm{t})\,\mathrm{d}t \tag{3}$$

The estimated expected income from historical sharing car rental data from November 2020 to June 2021 will be considered here and in the future. The standard deviation parameter for the Gauss filter used in the calculation of the expected income for the σ one-hour unit is $\sigma = 1.38$. The expected income in the late-night and early-morning hours close to zero is due to the fact that the rental of shared cars is closed between 1 AM and 5 AM.

Estimated forecasted incomes, for example, on Monday for four Riga city sectors are summarised in Figure 4. Data shows that there are significant differences between revenues for different sectors, as well as expected income changes by day. Moreover, some of the sectors can be very profitable only in specific time intervals and unprofitable in all others. For example, in the morning it is more convenient to move a car to sector D than to sector C, but after 1 PM a car in Sector C will be more profitable than a car parked in Sector D. These findings empirically indicate that it is essential to analyze how the expected daily income changes.

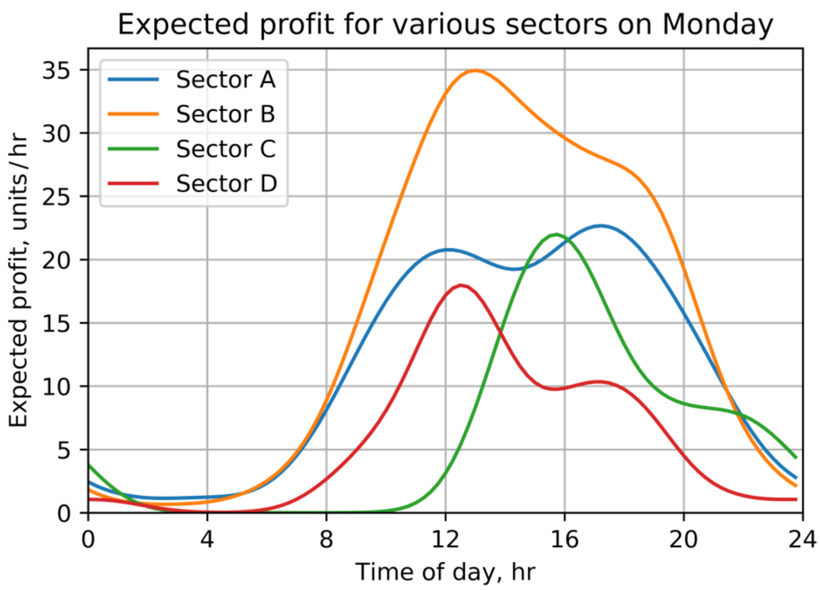

Figure 4. Expected income for the sectors on Monday

3.3.2 Weekly variations

There is a difference in estimated income for a particular sector between weekdays. From simple assumptions about the behaviour of sharing car users, it can be expected that demand for sharing cars, so expected income, could vary significantly between business days and holidays.

Indeed, such a phenomenon can be observed in estimated expected income for the sector A, as shown in Figure 5. Although there is a variation in expected income between business days, there is a very significant difference in expected income on holiday.

In other sectors, however, there is a more significant difference in expected income between other days of the week. For example, in the sector E (see Figure 6) incomes on Mondays and Sundays are slightly lower than in the rest of the week.

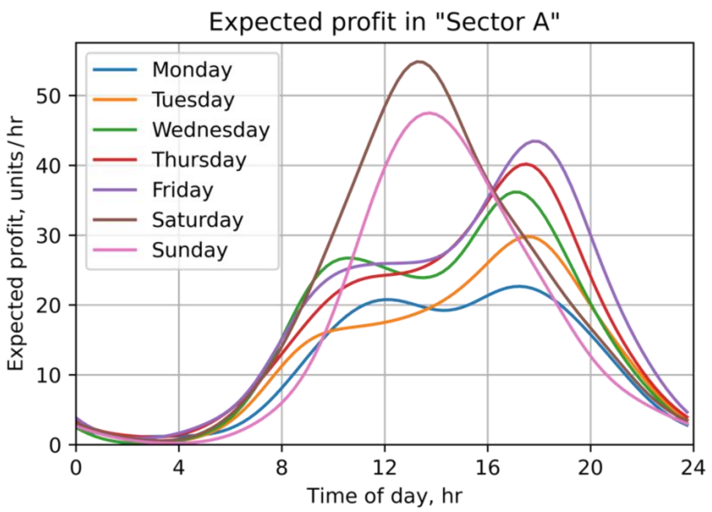

Figure 5. Expected income in “Sector A” in different days of week

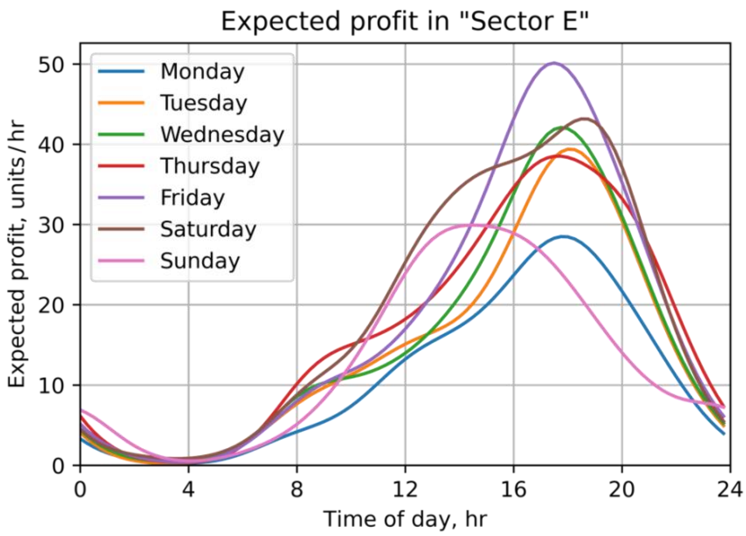

Figure 6. Expected income in “Sector E” in different days of week

As there is a significant but hard-to-predict difference between weekdays, it is necessary to calculate the expected earnings for each day of the week separately in the optimization algorithm.

3.3.3. Variation in Historical Data

A related subject matter related to the modelling of forecasted income from historical data is how old historical data is effectively used. Since the number of sharing cars is final, it may be considered necessary to use a maximum history that could reduce "noise" in the data obtained. However, as a counter argument for the use of historical data, it can be mentioned that user demand for shared cars is not static but is changing in result of seasonal or other long-term processes.

The following Figure 6 shows the estimated expected income from different periods of historical data (listed in the chart legend) in the sector B. These data show an increase in expected income when more recent data are used. This variation points to the need to find a balance between the amount of data used and the relevance of the historical data when calculating the expected income. It is also concluded that, in order to keep the calculation of the intended income used up to date, the expected income should be re-calculated on a regular basis.

At present, however, there is not enough historical data available to assess whether the change observed is due to fluctuations in seasonal demand or for any other reason.

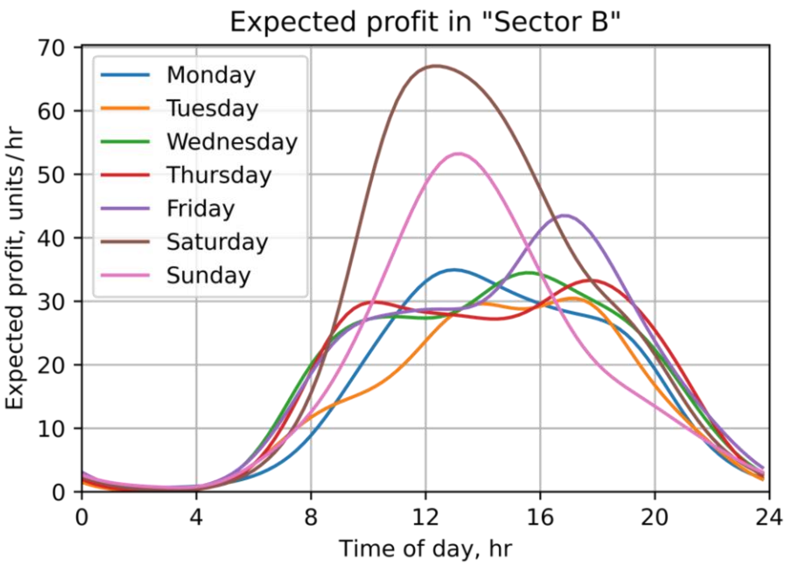

Figure 7. Expected income in "Sector B" on Monday

3.3.4. Limitations of the Proposed Method

While the method of estimating the expected income provides a valuable numerical estimate of the cost of parking sharing cars in specific sectors, effective use of the method described must be aware of its shortcomings and limitations. For example, the method described may provide inaccurate results if too many vehicles are placed in a particular sector, or the area of the sector is too small. Similarly, it is necessary to have a reliable history of bookings to estimate the expected income, and it may not be available when starting a sharing car operation in a new region.

The following chapters describe the three main shortcomings and limits for estimating of expected income.

3.3.5. Linearity Assumption

In the definition and calculation process described above, there is an assumption that the estimated income for a particular car parked in a sector does not depend on the total number of parked vehicles in that sector. At an extremely large number of cars parked in the same sector, you can see that this assumption is flawed; if the number of cars parked in the sector significantly exceeds the demand for shared cars in this sector, the average per car income will be very low.

Information collected from historical data (see Figure 8) allows you to analyse the veracity of the statement described. The graph shows the average number of rented cars in the sector A over two hours, depending on the number of parked cars in this sector.

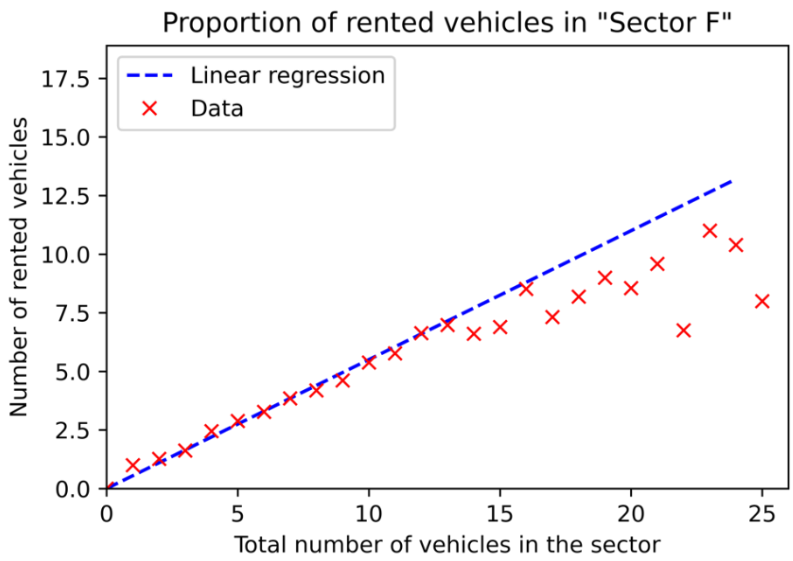

Figure 8. Proportion of rented cars in "Sector F"

The graph shows that the percentage of cars rented in this sector is almost constant at a small number of cars (55% of the cars located in the sector will be rented within two hours). Hence, the possibility of renting a particular car does not depend on the total number of cars in the sector at low number of available cars. And therefore, the expected per car income does not depend on the total number of parked cars in the sector.

However, the schedule shows that the possibility of renting a particular car over the next two hours is falling at a large number of cars. Accordingly, the estimated income for a large number of vehicles will not be up to date as average income per parked car will be lower.

The breakpoint in the sector A, when the described revenue model is ineffective, is around 14 cars. However, such a number of cars is higher than the typical number of cars in the sector A, so the described model for calculating the expected income is acceptable approximation. Similar data analysis and finding a breakpoint can be used to find the flexible demand limit for each sector.

3.3.6. Usage of Historical Data

The described method for calculating of expected income is based on the existence of historical data. Unlike demographic-based models, the described model cannot be used when starting a sharing car rental system in a new city, or by expanding operations into new sectors.

Reliance on historical data prevents a model from repid responding to changes in demand for shared cars, or to price policies. The described expected income estimation algorithm will work most accurately if variations in the shared car system are minimal.

3.3.7. Inaccuracies in Small Sectors

Small sectors may lack data to adequately calculate expected income due to data noise. This phenomenon limits the lower size of sectors, thereby affecting the constructing of sectors.

3.4. Car-sharing Income Optimization by E-vehicle Relocation

This chapter, unlike the description of the informally described substantive model in the previous chapter, will provide an exact e-vehicle sharing model, expressed in a set of appropriate parameters. This allows you to optimize service income when relocating e-vehicles.

This work is primarily inspired by the Mixed-Integer Programming (MIP) model proposed by (Gambella et al., 2018) for optimizing relocation operations in electric car-sharing. However, there are two major issues when utilizing this model in our case study—carsharing platforms in Riga, Latvia:

- Such model is built for a reservation-based system, where all the customer requests are made in advance and the exact demand for vehicles is known prior to the relocation operations. Therefore, this model does not work for system where the reservations are requested real-time.
- Solving the described model is rather time-consuming, and for sufficiently large operations (with a great number of service vehicles, discrete sectors, and relocators) the model is not solvable in any attainable time.

We propose a way to adapt the model, so that these two limiting factors are mitigated. The lack of prior reservations is solved by considering two other metrics instead: routine arcs (see Section 3.2) and predicted profits (see Section 3.3). The significant solution times for the model are addressed by not considering the path of each vehicle sepperately, but rather the collective paths of the swarm of similar e-vehicles instead.

3.4.1. Model Parameters

Let S be the set of all operating sectors, where each sector i ∈ S is characterized by the maximum number of electric vehicles (EVs) that can be stationed there - capacity C_i. Let H be the set of e-vehicles. Let B be the number of seats in each EV. Let H_x be a subset of H where all e-vehicles have initial battery charge above some predetermined level (e.g. 25 %), and let H_z be a subset of H where all EVs have initial battery charge under that level. Then $H_x \cup H_z = H$. Let H_{xi} denote the subset of H_x where all EVs in H_{xi} are initially stationed at station i. Let Q be the set of relocators, and Q_i be a subset of Q where all relocators in Q_i are initially stationed at station i. Let T be the set of discretized time instants over which the model operates: T = {0, 1, . . . , T_{max}}. Let A be the set of arcs over which vehicles and relocators can move in the time-space network S × T. Each arc is characterized by multiple parameters:

$$a = \{i_a, i'_a, t_a, t'_a, p_a, c_a, d_a\} \tag{4}$$

where:

- starting station and ending station for arc a is denoted by i_a and i'_a respectively,
- starting time and ending time for arc a is denoted by t_a and t'_a respectively,
- monetary profit for arc a is denoted by p_a, can be obtained from historical data,
- battery charge increase is denoted by c_a.
- customer demand for arc a is denoted by d_a.

The set of arcs A is divided into four subsets $A_w \cup A_t \cup A_t \cup A_c = A$,
where:

- A_w - set of “waiting arcs” over which relocators and EVs travel forwards through time and stay at the same station. An arc a ∈ A_w is characterized by $i_a = i'_a$, $t'_a = t_a + 1$, $d_a = 0$, $p_a \geq 0$, $c_a \geq 0$. A separate waiting arc a ∈ A_w is generated for each combination of starting stations i_a ∈ S and starting time instants t_a ∈ T.
- A_r - set of “relocation arcs” over which relocators and EVs travel together from one station to another. An arc a ∈ A_r is characterized by $i_a = i'_a$, $t'_a > t_a$, $d_a = 0$, $p_a = 0$, $c_a = 0$. A separate relocation arc a ∈ A_r is generated for each combination of starting stations i_a ∈ S, end stations i'_a ∈ S, and starting time instants t_a ∈ T.

- A_t - set of “transfer arcs” over which relocators are transferred from one station to another without the use of an EV. An arc $a \in A_t$ is characterized by $i'_a = i'_a$, $t'_a > t_a$, $d_a = 0$, $p_a = 0$, $c_a = 0$. A separate transfer arc $a \in A_t$ is generated for each combination of starting stations $i_a \in S$, end stations $I'_a \in S$, and starting time instants $t_a \in T$.
- A_c - set of “customer arcs” over which customers have requested to move (or have been predicted to move) with EVs. An arc $a \in A_c$ is characterized by $t'_a > t_a$, $d_a > 0$, $p_a > 0$, $c_a = 0$.

For shorthand in later sections, several other subsets of A are defined. Let A_x be the subset of A over which EVs $h \in H_x$ with significant battery charge can move: $A_x = A_c \cup A_w \cup A_r$. Let A_z be the subset of A over which EVs $h \in H_z$ with low battery charge can move: $A_z = A_w \cup A_r$. Let A_y be the subset of A over which relocators $q \in Q$ can move: $A_y = A_t \cup A_w \cup A_r$.

3.4.2. Variables

The following variables are influenced by the MIP model to try to maximize the objective. Let x_a be the number of vehicles $h \in H_x$ travelling over an arc $a \in A_x$.

Let z_a be the number of vehicles $h \in H_z$ travelling over an arc $a \in A_z$. Let y_a be the number of relocators $q \in Q$ travelling over an arc $a \in A_y$.

3.4.3. Objective

The objective for the model should be to:

- distribute EVs with significant battery charge $h \in H_x$ to the most profitable sectors or sectors from which customer requests originate;
- move EVs with low battery charge $h \in H_z$ to the charging stations.

Let α be the monetary value of a fully charged battery on an e-vehicle (this could be, for example, the expected profit from a fully charged e-vehicle over the course of its full discharge). Then the model objective is defined as follows:

$$\mathrm{M} = \max\left[\sum_{a \in A_x} x_a p_a + \alpha \cdot \sum_{a \in A_z} z_a c_a\right] \tag{5}$$

3.4.4. Constraints

The MIP model is defined to maximize the objective (5), so that the following constraints (conditions) are met:

Constraint (6) ensures that the customer demand associated with each arc is not exceeded.

$$x_a \leq d_a, \text{kur } a \in A_\text{c} \tag{6}$$

Constraint (7) imposes that the maximum number of vehicles that can be stationed at each sector is not exceeded.

$$x_a + z_a \leq C_{i_a}, \text{kur } a \in A_\text{w} \tag{7}$$

Constraint (8) guarantees that the number of fully charged vehicles at the initial time moment at each station is respected. Constraints (9) and (10) imposes the same restriction for vehicles with almost depleted battery and relocators respectively.

$$\sum_{a \in A'} x_a = H_{\text{x}i}, \text{kur } A' = \{a \in A_\text{x} \mid t_a = 0, i_a = i\}, i \in S \tag{8}$$

$$\sum_{a \in A'} z_a = H_{\text{z}i}, \text{kur } A' = \{a \in A_\text{z} \mid t_a = 0, i_a = i\}, i \in S \tag{9}$$

$$\sum_{a \in A'} y_a = H_{\text{y}i}, \text{kur } A' = \{a \in A_\text{y} \mid t_a = 0, i_a = i\}, i \in S \tag{10}$$

Constraint (11) imposes vehicle flow conservation at timespace network nodes, whereas constraints (12) and (13) impose flow conservation for vehicles with low battery and relocators respectively.

$$\sum_{a \in A^+} x_a = \sum_{a \in A^-} x_a, \text{kur } i \in S, t \in T, 0 < t < T_\text{max}, \begin{matrix} A^+ = \{a \in A_\text{x} \mid i'_a = i, t'_a = t\} \\ A^- = \{a \in A_\text{x} \mid i_a = i, t_a = t\} \end{matrix} \tag{11}$$

$$\sum_{a \in A^+} z_a = \sum_{a \in A^-} z_a, \text{kur } i \in S, t \in T, 0 < t < T_\text{max}, \begin{matrix} A^+ = \{a \in A_\text{z} \mid i'_a = i, t'_a = t\} \\ A^- = \{a \in A_\text{z} \mid i_a = i, t_a = t\} \end{matrix} \tag{12}$$

$$\sum_{a \in A^+} y_a = \sum_{a \in A^-} y_a, \text{kur } i \in S, t \in T, 0 < t < T_\text{max}, \begin{matrix} A^+ = \{a \in A_\text{y} \mid i'_a = i, t'_a = t\} \\ A^- = \{a \in A_\text{y} \mid i_a = i, t_a = t\} \end{matrix} \tag{13}$$

Constraint (14) imposes that the total number of vehicles travelling along an arc does not exceed the number of relocators travelling along the same arc. Constraint (15) guarantees that at most B relocators can travel in a vehicle (either as passengers or a driver).

$$x_a + z_a \leq y_a, \text{kur } a \in A_\text{r} \tag{14}$$

$$y_a \leq B \cdot (x_a + z_a), \text{kur } a \in A_\text{r} \tag{15}$$

Constraints (16), (17), and (18) define the vehicle and relocator variables used by the model.

$$x_a \in \{0, 1, \dots, |H_x|\}, \text{kur } a \in A_\text{x} \tag{16}$$

$$z_a \in \{0, 1, \dots, |H_z|\}, \text{kur } a \in A_\text{z} \tag{17}$$

$$y_a \in \{0, 1, \dots, |H_y|\}, \text{kur } a \in A_\text{y} \tag{18}$$

3.3.5. Rationale

The model proposed in (Illgen at al., 2019) operates on binary variables for each arc and vehicle pair, therefore its variable complexity is in the order of $2^{|H|\cdot|A|}$. As our proposed model does not have to keep track of precise battery charges for each vehicle at every given time moment, we can treat all vehicles as homogenous (apart from dividing them into two subsets Hx and Hx). As a result, our proposed model has the variable complexity in the order of $|H|^{|A|}$, which is smaller than $2^{|H|\cdot|A|}$, and, therefore, hopefully it would be solver faster:

$$|H|^{|A|} < 2^{|H|\cdot|A|} \quad \Rightarrow \quad \log_2|H| < |H|$$

As most arcs that customers would like to take are not requested or are not predictable prior to solving the model, we propose another way to measure the favorability of vehicle positions. Each waiting arc $a \in A_w$ is assigned a positive arc profit p_a which equates the expected profit for a vehicle located at the sector i_a at the time t_a. More on calculating the expected profit for each waiting arc see in next section.

3.5. Model evaluation

The model proposed in (Illgen at al., 2019) and our model were evaluated on a set of sample input system parameters. In this comparison, the input data was constructed so, that the battery charges for the e-vehicles would be ignored in both models (the solution time for model proposed in (Illgen at al., 2019) without ignoring battery charges is few orders of magnitude greater than when ignoring the battery charges). Also, in input data $A_t = \emptyset$. Models were solved for a maximum of 0,5 minutes on a 1.4 GHz, 8 GB machine using SCIP solver wrapper provided by Google OR-Tools (Vasconselos et al., 2017). The time τ for finding the best solution 3 in the given time moment was recorded together with the optimizer value. The results are summarized in the Table I. If the best-found solution was deemed as feasible, but not optimal, then a dagger (†) is displayed next to the solution time.

Table 1. (Gambella et al., 2018) algorithms comparison

SYSTEM'S PARAMETERS					THE NEW MODEL		THE ORIGINAL MODEL	
$\lvert S \rvert$	$\lvert H \rvert$	$\lvert Q \rvert$	$\lvert T \rvert$	$\lvert A_c \rvert$	τ, S	v, EUR	τ, S	v, EUR
5	10	2	8	10	0.0	8.76	0.1	8.76
10	10	3	12	30	0.1	26.28	2.2	26.28
10	10	3	24	30	†85.5	23.65	NO SOLUTION	
15	10	3	12	30	†54.8	26.93	†2.0	–27.73
10	20	3	12	80	†43.1	60.49	NO SOLUTION	
12	4	1	15	40	6.4	11.7	0.7	11.7
15	5	1	16	50	12.9	20.14	1.5	20.14
10	12	2	20	50	41.3	44.81	NO SOLUTION	
10	15	2	30	60	†75.9	42.63	NO SOLUTION	
18	20	3	10	80	†79.2	96.97	NO SOLUTION	
30	50	4	10	100	†3.2	171.51	NO SOLUTION	
30	60	5	12	120	†3.3	231.81	NO SOLUTION	

Both models were tested on the last two system parameters in Table I again, but with the time limit of 0.5 hours. Neither model found the optimal solution, and only our model found feasible solutions. After introducing battery constraints and "expected profit" from waiting arcs in our model, for realistically sized system parameters (see last row in Table I) a feasible solution could be found in under 10 minutes.

4. Research Findings

The proposed solution uses e-vehicle sharing historical data — the intensity of trips across different sectors of the city, depending on season, day of week and clock time. This data can be obtained by recording the sharing events that the service provider information system can manage. The division of the city into the sectors is also carried out using historical data, which in turn affects the permissible set of relocations and their costs. The model described is therefore applicable after the introduction of a sharing transport service and the accumulation of historical data.

It should be acknowledged that e-vehicle rental services can be organised in many ways. The company offering services in Riga, Latvia provides a dynamic optimization of e-vehicle relocation approach, which many cities do not offer. A more sophisticated service that would offer the use of one e-vehicle to many customers, such as joint trips, is not present in any city. Obviously, the service capabilities determine the complexity of the model and its effectiveness. The rapid development of IoT capabilities will offer ever-new service capabilities that will require ever-new solutions. This calls into question the possibility of a single, universal solution.

5. Conclusions

The study offers a model for the use of sharing vehicles, described as a commercial system with one-way trips and the dynamic relocations of e-vehicles between city sectors, without pre-booked trips. The model consists of the following set of parameters: breakdown of the city in sectors, maximal number of available cars, number of cars per sector, set of possible relocations, parameters characterising e-vehicle relocations, number of available relocators.

The parameters used in the model allow you to describe the operation of a real system:

- The strategic level determines the number of e-vehicle available for sharing and the maximum number of e-vehicle to be placed in each urban sector.
- At a tactical level, historical vehicle sharing data allows you to assess the profitability of e-vehicle depending on the season, day of the week, usage time and city sector in which e-vehicle is placed.
- At operational level, the total daily income is estimated at the sum of the average expected income over a specific period of time from all e-vehicle placed in a specific sector that can be rented by users.

The study provides an algorithm that optimizes expected income based on the set of selected relocations using the values of the above parameters. When implementing an algorithm, special attention should be paid to its performance as optimization must be performed dynamically, within few hour interval.

The vehicle sharing model proposed in the study should be considered as only one step in reaching an optimal solution. The model only partly describes real-life processes, such as e-vehicle can vary between battery capacity and technical parameters and prices. These parameters have not been taken into account in the given model, and their research may be the content of further studies. In future studies, the actual assessment of income generated by the relocation of e-vehicles is also relevant.

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