Baltic J. Modern Computing, Vol. 11 (2023), No. 1, 50-66 https://doi.org/10.22364/bjmc.2023.11.1.04

Entropy of video lecture

Linda ALKSNE¹, Lauris CIKOVSKIS², Andris OZOLS²

¹Liepaja University, Lielā Street 14, Liepāja, LV-3401, Latvia ²Riga Technical University, Kalku Street 1, Riga, LV-1658, Latvia

linda.alksne@liepu.lv, lauris.cikovskis@rtu.lv, andris.ozols@rtu.lv

Abstract. In this paper, the relation between video lectures Shannon and perception quality defined by number of known criteria (guidelines) of an optimal video lecture is studied. The entropy both for sound and video sub-channels are calculated using Matlab. The obtained results show that perception quality increases when the video entropy is decreased. It is also found that there is correlation between the type of the video lecture and video entropy.

Keywords: Entropy, information theory, Matlab, video lecture

1. Introduction

During pandemic video lectures have been delivered in different possible ways, teaching is happening online and how these lectures have been prepared and what students can see or hear during these lectures have been not so important as the lectures would happen anyhow. We may observe the lack of a common framework, model, and guidelines on how to shift from the offline mode to the online mode of teaching and learning processes. Every teacher has been putting his / her best efforts to carry out the remote teaching process, but the heterogeneity of the approaches can easily be observed (Kumar K., 2021). When the authors began to analyze video lectures as code, at that time there still were discussions about how e-learning is affecting studies, pros and cons, and also the quality of video lectures, and how it would be possible to deliver lectures so students could perceive information better. There have been different studies about what kind of video lectures give students better results, and they have been collected in paper by L.Alksne "How to produce video lectures to engage students and deliver the maximum amount of information" (Alksne, 2016). It should be remembered that lectures included in this article were analyzed only according to their technical parameters.

The paper aims to find correlation between the quality of video lectures and entropy of them, and to continue to understand how to capture video lectures to achieve maximum information delivered.

Entropy as the measure of information has been used in ETL tools for business intelligence (Balta, 2007), as the measure of safety for passwords (Burnett, 2006) and also for making decisions (Lopez, 2007).

Paper consists of theoretical and analytical parts. Theoretical part (section 2) describes methodology how to calculate entropy of video lectures with matlab. In section 3 authors analyze the results of video and audio entropies of chosen video lectures.

2. Methodology

There were 11 different video lectures chosen. All different from each other. Each video was compared to the guidelines for producing video that engages students the most. From paper mentioned above L.Alksne "How to produce video lectures to engage students and deliver the maximum amount of information" guidelines that applies to published video was taken:

• the voice-over presentation type video lecture attracts the highest sustained attention;

instructor can make good eye contact

• strong presentation of relief and change-of-pace elements are most effective for learning;

• the layout colors should not be too diversified;

• Khan-style tutorial videos are more engaging than PowerPoint slides and/or code screencasts;

• text should be written in clear handwriting and good drawing skills should be used;

• 160 words per minute are recommended as the optimum speaking rate for presentations;

• capture videos in well-lit areas; the subject of the videos should be easily discernible;

all shots should be clearly focused and well-framed;

• zooming should be used only for focusing attention and should otherwise generally be avoided;

• keep the camera as still as possible;

• the sound should be audible and clearly understood throughout the video, with minimal to no ambient noise.

• adding the text to the video, choose a standard font.

• effects: Fade ins/fade outs, etc., should not detract from the message of the video. They should be smooth and not abrupt and choppy.

• Dead air: Edit out when possible. Background noise: Background noise, breaks, skips, hissing should be edited out.

• if adding background audio intentionally, balance with primary audio as best as possible;

• extensive introductory material should be edited out and included in the written description.

These guidelines have been mentioned by different authors such as (Dai, 2012), (Bennet, 2007), (Chen, 2011 and 2015) and (Williams, 1998), and also internet platforms such as (Wistia, 2013) and Youtube. Technical rules are published by Association for recorded sound collections (WEB, a).

3. Selection and analyses of video lectures

All the video lectures were analyzed to see how they meet the guidelines. The results are summarized in the Table 1.

Video No.1 (Fig. 1) is a video excerpt from Zanda Rubene's lecture. It was chosen because there is nothing that could be an obstruction while watching a video lecture. Also, the lecturer is speaking at a normal pace and convincingly, there is no unforeseen action or image change during the lecture. When the authors subjectively evaluated this lecture, it seemed easy to perceive, so it was chosen for analysis according to the guidelines and entropy calculations.

The videos that were chosen was approximately 10 minutes long, and matlab counted frames for each video. In the pictures the frame when the screenshoot from the video was taken can been seen and also the total amount of frames in the video.



Fig. 1. Video lecture of Zanda Rubene https://www.youtube.com/watch?v=MN-xCEdEDJI&t=56s

Video No.2 (Fig. 2) is a video excerpt from Juris Blūms lecture. This lecture was chosen because there are no effects or editing used. It is a natural lecture from the classroom, captured during the lecture with students.



Fig. 2. Video lecture of Juris Blūms. https://www.youtube.com/watch?v=85CNYz3uyVs&t=520s

Alksne et al.

Video No.3 (Fig. 3) is a Khan style video lecture excerpt from Julija Maksimkina lecture. This lecture was chosen because the guidelines say that Khan style video lectures students find it easiest to perceive. Also, Khan style videos are very popular, so it seemed very interesting to analyze such a type of video. Khan-style tutorials require more pre-production planning than presenting slides or typing a code into a text editor. The most effective Khan-style tutorials are those made by instructors with a clear handwriting, good drawing skills, and careful layout planning so as not to overcrowd the canvas. It has been recommended to record Khan-style tutorials when possible (Williams, 1998).

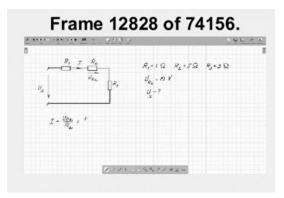


Fig. 3. Video lecture of Julia Maksimkina https://www.youtube.com/watch?v=SFnf4RK3uuk&t=994s

Video No.4 (Fig. 4) is a video excerpt from Aleksandrs Dolgicers lecture. This lecture attracts attention because it is filmed from two cameras at the same time. The room is a lab that is full of different things. At the same time, a wide variety of information is transmitted, and the sound and video quality is mediocre.

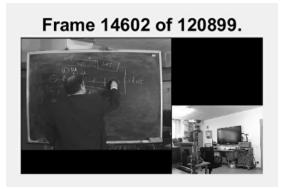


Fig. 4. Video of Aleksandrs Dolgicers. https://www.youtube.com/watch?v=8QAYVyqAoxs&t=884s

Video No.5 (Fig. 5) is a video excerpt from Ingus Skadiņš lecture. This video has been chosen because it changes the environment in which it is filmed, both in the auditorium and at the

54

whiteboard, as well as using writing and drawing on the whiteboard. Although it is similar to video no.4, it has good sound and video quality.

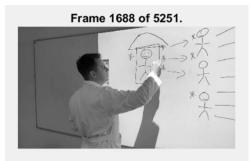


Fig. 5. Video lecture of Ingus Skadiņš. https://www.youtube.com/watch?v=baz9LYvWsro

Video No.6 (Fig. 6) is a video excerpt from Ansis Jurgis Stabingis lecture.

This lecture is very similar to lecture video no.1, only the background changes and the fact that the teacher is a man.



Fig. 6. Video lecture of Ansis Jurgis Stabingis https://www.youtube.com/watch?v=76lpS1hHbfY

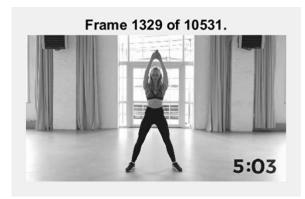


Fig. 7. Video of Paula Freimane. https://www.youtube.com/watch?v=aZbfWLezvlk

Video No.7 (Fig. 7) is a fitness training video from Paula Freimane. This video is completely different. It's a sports workout, but it's also filmed on purpose as a video workout, where the viewer should be able to capture the video right away because they have to do everything at the same time as the instructor's video.

Video No.8 (Fig. 8) is a video lecture where different experiments are captured and explained. It is from a public course in physics by Riga Technical University. This video was chosen because of different effects and also because of various information that changes very fast during the video.

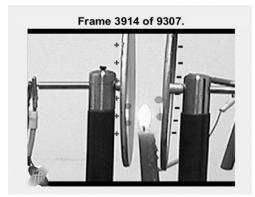


Fig. 8. Experiment in physics.

Video No.9 (Fig. 9) is a video lecture from Inta Volodko. From a guidelines point of view this is a very good lecture that meets almost all the criteria for being a very perceptible video lecture. It was a pleasure to find that something like this video lectures also is possible to capture here in Latvia. It has been proven that the voice-over presentation type generates the highest sustained attention (Chen, 2015).



Fig. 9. Video lecture of Inta Volodko. https://www.youtube.com/watch?v=ZOuuaLXXCI4&list=PLoze_Ym7-r3_ooCHJIWQPFy3FolQTiqK&index=1

Video No.10 (Fig. 10) is a video lecture from professor Andris Ozols. This lecture is currently being experienced by students and pupils around the world as universities and schools

Entropy of video lecture

have switched entirely to distance learning. Various video conferencing tools are used to provide these lectures, in this case Zoom. The lecture shows both the lecturer and the students, as well as various rooms, pictures, as well as the screen is shared. Professor in the video also speaks fast during lecture and students generally engage more in videos where instructors speak faster. Some practitioners recommend 160 words per minute as the optimum speaking rate for presentations (Williams, 1998).

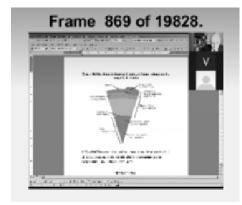


Fig. 10. Video lecture of Andris Ozols.

Video No.11 (Fig. 11) is the same lecturer and form of lecture as in video No.1 And only the background is changing.



Fig. 11. Video lecture of Zanda Rubene. https://www.youtube.com/watch?v=aaWgm0JBrjo The results are displayed together with the first video. Alksne et al.

4. Evaluation of entropy in Matlab

Natural lecture delivered by a lecturer is treated as a noiseless communication channel consisting of a sound sub channel and light sub channel. Each sub channel is transmitting frames whose Shannon entropies are calculated according to formula (1)

$$E = \frac{\sum_{n=1}^{N} p(n) \log_2 p(n)}{\log_2 N} \tag{1}$$

where p - probability distribution,

 $N-\ensuremath{\text{the total}}$ number of points in the distribution.

The entropy value E is normalized to obtain a relative measure that could be compared among video lectures. log2N in the formula (1) represents maximal entropy.

- For each video lecture three different entropies are calculated:
- a) entropy of video frames (video entropy)
- b) entropy of audio signal intensity (audio temporal entropy)
- c) entropy of audio signal spectrum (audio spectral entropy)

Because video lectures have slow changing scenes, entropy is calculated not for every frame but for a smaller number of randomly selected frames. We assume that these frames capture enough information to characterize the whole lecture. Audio frames/samples are selected in the same manner. Finally average entropy level is obtained which is

$$E = \frac{\sum_{m=1}^{M} E(m)}{M}$$
(2)

where M – number of video or audio frames.

Next, we explain the algorithm of the Matlab program for entropy calculation.

4.1. Video entropy

- 1. Video data is read from the video lecture file.
 - videoObject = VideoReader(movieFullFileName);
- 2. M random frames are selected. Our results are obtained using 250 frames for each lecture. For a 5 minutes long sample it is 50 frames/minute.
- 3. Each frame from m=1 to M is analysed
- 4. RGB color frame is converted to grayscale image.
 grayImage = rgb2gray(thisFrame)
 - a. The histogram of the grayscale frame is obtained which contains the tonal distribution of the frame – number of pixels for each tonal value. A grayscale pixel is characterized by values between 0 and 255. h=imhist (grayImage, 256)
 - b. The histogram is normalized to obtain tonal probability distribution p=h./sum(h+1e-12);
 - c. Entropy is calculated using formula (1). E=-sum(p.*log2(p))/log2(length(p))

58

4.2. Audio temporal entropy

1. Audio track is read from the video lecture file. In the case of stereo sound only a single channel is used.

```
[y,Fs] = audioread(movieFullFileName);
y=y(:,1);
```

- 2. In a similar way as for video analyses (section A), M random audio frames are selected. Audio frame (also called window) length is chosen 214 samples which at 44,1 KHz sampling rate is equal to 0.372 seconds. Such length is approximately equal to the length of a single word when speech rate is 160 words per minute.
- 3. For each frame from m=1 to M is analysed
 - a. The histogram of sound intensity is obtained which contains the number of occurrences of each intensity level. Results are obtained with resolution 100 intensity levels. h=hist(audioF, 100)
 - b. The histogram is normalized to obtain a probability distribution for each intensity level. p=h./sum(h+1e-12)
 - c. Entropy is calculated using formula (1).

4.3. Audio spectral entropy

The initial steps are similar to the steps 1-2 of the analyses of temporal entropy (section B). Next for each frame from m=1 to M spectral entropy is calculated.

- Signal spectrum is obtained by using Fast Furrier transform:
 X=fft(audioF);
- b. Power spectral density is calculated S(n)~|X(n)|² X=X(1:winSize/2+1); S=abs(X).^2*(1/winSize^2); S(2:end-1)= 2*S(2:end-1);
- c. The power density is normalized to obtain power probability distribution. p=S./sum(S+1e-12);
- d. Entropy is calculated using formula (1). Scaling factor log₂N represents the maximal spectral entropy of the white noise.

As an alternative Matlab built-in function *entropy* could be used.

5. Results and analysis

All the results have been displayed in the Table 1 below. Further we assume that the quality of video lecture is proportional to the number of guidelines taken from literature. But let us discuss guidelines.

First guideline is the voice over presentation. There are three videos where we can do voice over presentations – it is where you do not see the speaker, but just see the video of slides and hear the voice of the teacher. Video No.3 is Khan style presentation, video No.8 is experiments and video no.10 is zoom video where part of it is voice over presentation. As you can see from

the Table 1 with results, those 3 lectures really have the lowest entropy. In this bottom score also is video no.9 but if we analyze the lecture, even if we can see the teacher, most of the screen presentation is displayed. So, the first guideline is approved – voice over presentation has the lowest structural information.

The second guideline is good eye contact with the teacher. In this category we can compare only video lectures where we can see the teacher. Those are: No's 1, 11, 5 and 9. From the results we can tell that we cannot detect if the teacher is looking straight in the camera or has been captured from the side if he does not move during the lecture. If the teacher is moving – you can see in video no.5 how the entropy changes while the teacher draws on the blackboard and when looking into the camera, then it is possible to detect that. So went counting YES for guidelines, this should be taken out or with zero coefficient. The same with the clear handwriting and drawings. This also is not possible to detect with entropy if we do not analyze only the entropy of handwriting. But this is a guideline which should be taken seriously for teachers. Standard font also is counted in this group.

Third has strong presentation, with change-of-pace elements. Videos no.2,3 and 9 has presentations as slide shows. If we look at the results of video entropy, then we can see that videos no.3 and no.9 really have low entropy. Video no.2 entropy is higher. If we analyze the video, we can see that there are different things shown in the video besides the presentation – there are two different teachers, furniture of auditory, and the focus also changes during presentation. So, we assume that if there is a strong presentation that does not have many side effects, then the entropy is lower – and for students it is easier to concentrate on presentation and teacher and not the other things in auditory. Also, we see that videos no.3 and 9 are captured as study materials, but the lecture no.2 is captured in auditory. It is easy to conclude that lectures captured in auditory will have higher entropy as they are not edited and there is no background or studio involved in the capturing process.

Layout colors are not too diversified and in well-lit areas – to find correlation between entropy and those guidelines, we should have two identical video lectures and to change only the light or the background. At first when videos No.1 and no.11 was chosen as very similar, but the background was only darker in video no.2 authors thought, this could be the way to check this theory, but video no.1 have higher entropy because the corner of the presentation is appearing in and out the screen, so for this guideline should be more experimenting with video capturing needed. But as it is a very important rule, this guideline will count as YES in video analysis.

Khan-style tutorial video there is only one and with the lowest entropy, so this guideline is true, it contains less Shannon information. Background audio balance is only for this training video, usually it is not used in video lectures – music all the time, so this guideline does not count as Yes.

Extensive introductory material – If we watch how entropy changes when there is introductory material before the lecture, we also can see how the entropy is changing in this area, also it changes the average entropy. Of course, it is necessary and convenient that we see this information, but if we discuss it from the technical side, it is better if the introduction is outside the video, maybe underneath it in the description where the lecture is published. But as this is normal for any video at all, we can consider not to count this as YES when counting all the guidelines that contain every video.

	No.1	No.2	No.3	No.4	No.5	No.6	No.7	No.8	No.9	No.10
the voice-over presentation	No	Yes	yes	no	no	no	no	no	Yes	No
good eye contact	Yes	No	no	no	yes	no	no	no	Yes	No
Strong presentation, change-of-pace elements	No	Yes	yes	no	no	no	-	no	Yes	No
layout colors not too diversified	Yes	Yes	yes	no	yes	no	yes	yes	Yes	No
Khan-style tutorial video	No	No	yes	no						
clear handwriting; good drawings	-	-	yes	No	yes	-	-	yes	-	-
100 words per minute	99	117	94	40	76	146	-	55	104	116
in well-lit area	Yes	No	yes	No	yes	no	yes	yes	Yes	Yes
subject of the videos is easily discernible	-	-	-	-	-	-	-	-	-	-
clearly focused and well-framed	Yes	Yes	yes	No	yes	yes	yes	yes	Yes	Yes
zooming should be avoided	Yes	No	yes	No	yes	yes	no	no	Yes	Yes
still camera	No	No	yes							
sound clearly understood	Yes	No	yes	no	yes	yes	yes	yes	Yes	Yes
minimum noise	Yes	No	Yes	no	yes	yes	yes	yes	Yes	No
standard font	Yes	Yes	No	no	no	yes	yes	yes	Yes	Yes
Smooth effects	Yes	Yes	Yes	no	yes	yes	no	yes	Yes	Yes
Background noise, breaks, skips, hissing should be edited out	Yes	No	Yes	no	yes	no	yes	yes	Yes	No
background audio balance	-	-	-	-	-	-	yes	-	-	-

Table 1. Number of guidelines followed and entropy results

extensive introductory material	Yes	No	Yes	no	yes	yes	yes	no	Yes	No
Not captured in lecture	No	No	Yes	Yes	Yes	No	Yes	Yes	Yes	Yes
Lector voice men/woman	Woman	man	woman	man	man	man	music	woman	woman	man
Average video entropy	Video no.1 0,8635 Video No.11 0,8516	0,8625	0,4794	0,6691	0,7492	0,9028	0,8289	0,7403	0,6589	0,4067
Average sound temporal entropy	Video No.1 0,748 Video No.11 0,754	0,822	0,759	0,791	0,810	0,747	0,893	0,459	0,743	0,777
Average sound spectral entropy	Video No.1 0,554 Video No.11 0,559	0,554	0,518	0,610	0,512	0,543	0,460	0,506	0,520	0,452
YES video	5	4	9	2	7	4	5	6	8	5
YES sound	4	2	4	0	3	3	4	3	5	2
Score	9	6	13	2	10	7	9	9	13	7

As we do not evaluate the content of video lectures, but just Shannon information, so we cannot use this guideline: subject of the videos is easily discernible.

Zooming should be avoided - this guideline we cannot see while looking at the results of average entropy, but we can clearly see it in the visual material of videos nr.2 (Fig. 12) and no.7 (Fig. 13) We can see how entropy and information changes while zooming the video. In video no.2 it happens once, but video nr.7 uses zooming all video long. It is also assumed it works that way if you do not have a still camera. As in video no.1 where this corner of presentation comes in and out of the video.

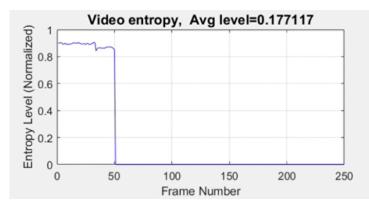


Fig. 12. Zooming effect of video no.2 (Fig. 2) to average video entropy

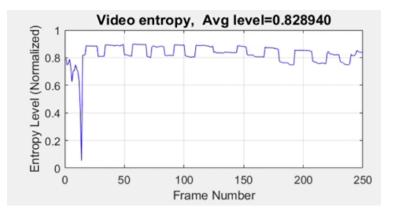


Fig. 13. Zooming effect of video no.7 to average video entropy

Smooth effects also we can include when analyzing a video lecture, because when the change of effects as zooming is not smooth, then the change of entropy is higher. If we count those guidelines that we can detect with entropy and get the results from average entropy, we can see that - the more guidelines are covered, the lower the entropy - so the Shannon information is diminishing.

Alksne et al.

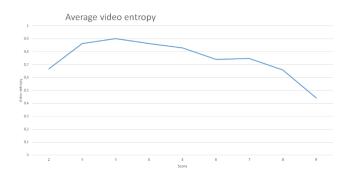


Fig. 14. Average entropy vs the number of YES (video compliance guidelines)

Of course, there are exceptions, in this case - video nr.4, as we see the video lecture gets only two YES, but even as there are two screens, there are also black parts of screen which does not change all the video long, so the entropy is low for this video.

To make some conclusions about sound entropy - we can see from the Table 1 that where the teacher is women, their entropy is lower, and the authors counted words for the first minute of the experiment, to get some description of the speaker. We cannot take this to make some decisions about the whole lecture, because, from lectures no.4 and no. 5 we can see that the teacher speaks more slowly when drawing and faster when speaking on camera. But we can see that in video no. 8 the entropy is really low, because the woman speaks slowly throughout the entire video.

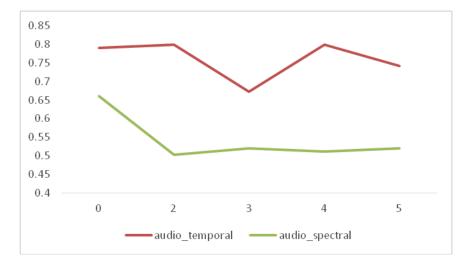


Fig. 15. Average entropy vs the number of YES (audio compliance guidelines)

From Fig.15 Average entropy vs the number of YES (audio compliance guidelines) we can see that there is no significant correlation between compliance with audio guidelines and audio spectral and temporal entropy, but the spectral and temporal analysis of each video lecture gives us great benefits, because we can draw conclusions about the effect of the speaker's / teacher's voice and speech on entropy.

6. Conclusions

- voice over presentation and Khan style video lectures have the lowest entropy;
- zooming videos, variable camera focus and smooth effects changes entropy;
- edited lectures, captured in studio have lower entropy;
- for lectures filmed in well-lit area and when layout colors are not too diversified, entropy is lower;
- there are guidelines that is not possible to detect with the changes of entropy;
- if the speaker is slower fluctuations of the entropy is bigger;
- if the speaker is slower average entropy is lower;
- entropy of video lectures with women voice is lower;
- videos with introduction screen have lower average sound entropy;
- entropy is higher when voice is faster and louder

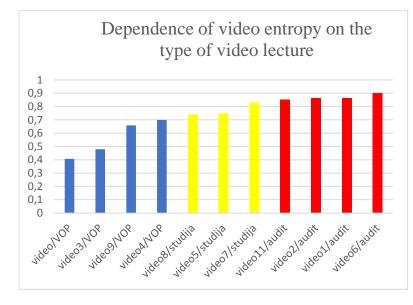


Fig. 16. Dependence of video entropy on the type of video lecture

Fig.nr.16 shows that the entropy of a video lecture depends on its type. With the lowest entropy, which also proves one of the guidelines, is Voice over Presentation (VOP). This group contains both Zoom videos and videos where you can see the instructor with the presentation. In the middle, with yellow colour, are those video lectures filmed according to the script and edited footage, without an audience. All lectures filmed in the auditorium have the highest entropy. From this, we can conclude that a student, watching a video lecture consisting of a presentation and the teacher's voice, will be able to understand the video much more easily than the video lectures filmed in the audience.

There are video lecture parameters that the instructor or the creator of the video lecture can change to affect the entropy - both through their behavior and through technical parameters. Some parameters need to be tested to demonstrate their effect on entropy by changing just one parameter in the video. For example, if there are no changes in the video, but the instructor only changes his speaking speed - he speaks faster or louder.

The main conclusion of this article is that there is a correlation between lecture type and entropy. Kahn-style video lectures with voice presentation have the lowest entropy, and studies have also shown that these lectures are easier for students to understand. (Chen, 2015)

Of course, this type of lecture cannot be used in all situations and it should not be done, the instructor can make the right decision when necessary - based on the knowledge of the type of video lecture and entropy.

Acknowledgements

The publication and presentation is made with the financial support of the project "Promotion of research, innovation and international cooperation in science at Liepaja University", Project No. 1.1.1.5/18/I/018.

References

- Alksne, L. (2016). How to produce video lectures to engage students and deliver the maximum amount of information. In SOCIETY. INTEGRATION. EDUCATION. Proceedings of the International Scientific Conference, 2, 503-516.
- Balta M. Data verification in ETL processes Romania, Alexandru Ioan Cuza University, 2007.
- Bennett, E, Maniar, N. (2007). Are videoed lectures an effective teaching tool?, 1–7 Brecht, H. D. (2012). Learning from online video lectures. Journal of Information Technology Education., 11, 227-250.
- Brecht, H. D., Ogilby, S. M. (2008). Enabling a Comprehensive Teaching Strategy: Video Lectures. *Journal of Information Technology Education*, 7, 71–86.
- Burnett, M., Kleiman, D. (2006) Perfect Password Selection, Protection, Authentication. Elsevier Science, pp.1-10.
- Dai, W., Fan, L. (2012). Discussion about the Pros and Cons and Recommendations for Multimedia Teaching in Local Vocational Schools. *Physics Procedia*, 33, 1144–1148.
- Chen, C.-M., Wu, C.-H. (2015). Effects of different video lecture types on sustained attention, emotion, cognitive load, and learning performance. *Computers & Education*, 80, 108–121.
- Google. YouTube Analytics. Available from Internet http://www.youtube.com/yt/playbook/ytanalytics.html#details
- Kumar K., Pande B.P. (2021) Rise of Online Teaching and Learning Processes During COVID-19 Pandemic. In Khosla P.K., Mittal M., Sharma D., Goyal L.M. (eds) Predictive and Preventive Measures for Covid-19 Pandemic. Algorithms for Intelligent Systems. Springer, Singapore
- Lopez Herrera J. Shannon entropy as a measure for making decisions. Spain, Universitat Politecnica de Catalunya, 2007.
- Shannon, C.E. (1948). A Mathematical Theory of Communication. The Bell System Technical Journal, Vol. 27, 1948. pp. 379–423, 623–656.
- WEB (a). Association for recorded sound collections. http://www.arscaudio.org/ committees/video_production_guidelines.html
- Williams, J. R. Guidelines for the use of multimedia in instruction. *Proceedings of the Human Factors and Ergonomics Society Annual Meeting* 42, 20, 1998. 1447–1451.
- Wistia (2013). Does length matter? It does for video! Available from Internet http://wistia.com/blog/does-length-matter-it-does-for-video

Received February 16, 2022, revised January 29, 2023, accepted February 25, 2023

66