Baltic J. Modern Computing, Vol. 12 (2024), No. 4, 434-442
https://doi.org/10.22364/bjmc.2024.12.4.05

Seismic Moment Tensor and Focal Mechanism of the M_W3.3 Earthquake of May 11, 2021 in the Kyoto-Osaka Border Region Determined by Waveform Inversion

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Abstract. The focal mechanism solution mainly depends on the number of stations used and becomes problematic especially in the case of weak earthquakes and sparse networks. In this study, we selected a weak crustal event and retrieve the seismic moment tensor of the M3.6 earthquake on 11 May 2021 (06:08:44.03 UTC, 34.836°N, 135.616°E, depth 12.49 km) in the Kyoto-Osaka border region determined by waveforms inversion from its records at only limited number of seismic stations. The seismic moment tensor was determined using only direct P- and S- waves separately at four stations and eleven stations of a local strong-motion seismological observation network. Our seismic moment tensor inversion is based on the point source approach and the use of only direct waves calculated by the matrix method. Displacements on the surface of an elastic, horizontally-layered medium are generated using the frequency-wavenumber integration technique. The advantage of using only direct P- and S- waves in our inversion method is that they are less sensitive to path effects compared to reflected and converted waves, which reduces the impact of an inaccurate velocity structure and improves the accuracy and reliability of the result. Based on the forward modeling, a numerical technique was developed for the inversion of the observed waveforms for the components of the moment tensor M(t) using the generalized inversion solution. The resulting solutions of the mechanism compare well with a very reliable version previously determined at three broadband stations by specialists of the Japanese National Research Institute for Earth Science and Disaster Resilience (NIED), which only confirms reliability of our inversion method and the very possibility of obtaining the useful results from data of only limited number of stations.

Keywords: focal mechanism, waveform inversion, seismic moment tensor, direct waves, seismic stations, earthquakes.

1. Introduction

Determining parameters of an earthquake source is an important task for a deeper study of the source processes itself, the calculation of theoretical seismograms at defined sites, a comparative analysis of different approaches in forward and inverse problems, and for the tasks of engineering seismology as well. In the case of weak-to-moderate earthquakes, the polarities and amplitudes of the first arrivals of P-waves observed at seismic stations are usually used to determine the focal mechanism. When there are available observed waveforms at a large number of stations with azimuths evenly distributed around the epicenter and a sufficiently accurate velocity model, the inversion usually results in a satisfactory and precise focal mechanism solution. The problem tackled in this contribution is an inversion of the focal mechanism if a low number of seismic stations is available. Also, accurate location of the source had to be known, as well as an adequate velocity model between the source and the stations, for calculation of emergence angles of first P-waves from the source. Usually, a sufficient number of reliable polarities is only available for large (M>4) earthquakes, occurring in the areas with dense seismological networks (Dziewonski et al., 1981; Arvidsson et al., 1998; Herrmann, 2002, 2008; Herrmann et al., 2008; Zhu et al., 2006). Utilizing of waveforms, containing much more information about the source than merely station polarities, can enable to overcome these limitations and using of fewer stations while determining the mechanisms of smaller earthquakes (D'Amico, 2014; Dreger & Helmberger, 1993; Malytskyy, 2010, 2016; Malytskyy & D'Amico, 2015). This is especially important in in seismic regions of Ukraine where local seismicity is relatively low and large earthquakes occur rarely. Addressing the problem of unavoidable inaccuracy of seismic waves modelling we propose to invert only the direct P- and S-waves instead of the full wavefield. An advantage of inverting only the direct waves consists in their much lesser distortion, if compared to reflected and converted waves, by inaccurate modelling of velocity contrasts, the direct waves carrying consequently much less distorted imprint by the source. An advantage, in this connection, of choosing the matrix method for calculation of the wave field consists in its ability to analytically isolate only the direct waves from the full field. Thus, for testing the proposed method, we chose the M3.6 earthquake on 11 May 2021 (06:08:44.03 UTC, 34.836°N, 135.616°E, depth 12.49 km) in the Kyoto-Osaka border (Japan), for which the seismic moment tensor solution or focal mechanism is quite precisely defined by the National Instutite for Earth Science and Disaster Resilience
https://www.fnet.bosai.go.jp/event/tdmt.php?_id=20210511060700&LANG=en).

2. Theory: waveform inversion

The method presented here enables obtaining of focal mechanism solution by inversion of waveforms recorded at only limited number of seismic stations. The inversion scheme consists of two steps. First (forward modeling), propagation of seismic waves in vertically inhomogeneous media is considered and a version of matrix method for calculation of synthetic seismograms on the upper surface of the horizontally layered isotropic medium is developed. The point source is located inside a layer and is represented by seismic moment tensor. The displacements on the upper surface are presented in matrix form in frequency and wave number domain, separately for far-field and near-field (Malytskyy, 2010, 2016). Subsequently, only the far-field displacements are considered and the wave-field from only direct P- and S-waves is isolated with application of eigenvector analysis reducing the problem to system of linear equations (Malytskyy, 2016). Subsequently (inverse modeling), spectra of the moment rate tensor components are calculated using a solution of generalized inversion and transformed to time domain by applying the inverse Fourier transform.

Assuming that the point source, represented by symmetric moment rate tensor **M**(t), is located within a stack of solid, isotropic, homogeneous, perfectly elastic layers with horizontal and perfectly contacting interfaces, the layers characterized by thickness, density, and velocities of P- and S-waves, the following expressions have been obtained by Malytskyy (2010, 2016) in cylindrical coordinates for the displacements $u_z^{(0)}(t,r,\varphi)$, $u_r^{(0)}(t,r,\varphi)$ and $u_\varphi^{(0)}(t,r,\varphi)$ on the upper surface of the half-space at z=0:

$$\begin{pmatrix} u_z^{(0)} \\ u_r^{(0)} \end{pmatrix} = \sum_{i=1}^{3}\int_0^\infty k^2 \mathbf{I}_i L^{-1}\left[m_i \mathbf{g}_i\right] dk \ , \quad u_\varphi^{(0)} = \sum_{i=5}^{6}\int_0^\infty k^2 J_i L^{-1}\left[m_i g_{\varphi i}\right] dk \ , \tag{1}$$

where

$$\begin{gathered} m_1 = M_{xz}\cos\varphi + M_{yz}\sin\varphi\ , \quad m_2 = M_{zz}\ , \\ m_3 = \cos^2\varphi\cdot M_{xx} + \sin^2\varphi\cdot M_{yy} + \sin 2\varphi\cdot M_{xy}\ , \\ m_4 = -\cos 2\varphi\cdot M_{xx} + \cos 2\varphi\cdot M_{yy} - 2\sin 2\varphi\cdot M_{xy}\ , m_5 = M_{yz}\cos\varphi - M_{xz}\sin\varphi\ , \\ m_6 = \sin 2\varphi\cdot M_{xx} - \sin 2\varphi\cdot M_{yy} - 2\cos 2\varphi\cdot M_{xy}\ , \end{gathered} \tag{2}$$

M_{xx}, M_{xy},…, M_{zz} are the Cartesian components of the moment rate tensor **M** in the frequency domain representing the source located at r=0, axis x pointing North and y East, φ is the station's azimuth, k is the horizontal wave number, functions $\mathbf{g}_i = \left(g_{zi}, g_{ri}\right)^T$, and $g_{\varphi i}$ contain propagation effects between the source and the receiver,

$\mathbf{I}_1 = \begin{pmatrix} J_1 & 0 \\ 0 & J_0 \end{pmatrix}$, $\mathbf{I}_2 = \begin{pmatrix} J_0 & 0 \\ 0 & J_1 \end{pmatrix}$, $\mathbf{I}_3 = \mathbf{I}_2$, $J_5 = J_0$, $J_6 = J_1$ are the Bessel functions of argument kr, and L^{-1} is the inverse Laplace transform, from frequency to time domain.

Further, only the far-field displacements are considered and the wave-field from only direct P- and S-waves is isolated with application of eigenvector analysis reducing the problem to a system of linear equations (Malytskyy, 2016). Eq. (1) is then expressed in matrix form for only the direct P- and S-waves on the upper surface of the half-space in frequency and wave number domain (ω, k) (Malytskyy, 2010):

$$\mathbf{U}^{(0)} = \mathbf{K}\cdot\boldsymbol{M} \tag{3}$$

where vector $\mathbf{U}^{(0)} = \left(U_x^{(0)P}, U_x^{(0)S}, U_y^{(0)P}, U_y^{(0)S}, U_z^{(0)P}, U_z^{(0)S}\right)^T$ contains the six Cartesian displacement components of direct P- and S-waves, vector

$\boldsymbol{M} = \left(M_{xz},\ M_{yz},\ M_{zz},\ M_{xx},\ M_{yy},\ M_{xy}\right)^T$ consists of the six independent Cartesian components of moment rate tensor **M**, matrix **K** accounting for path effects and transformations between the Cartesian and cylindrical coordinates. A least-squares solution to the over-determined system of eq. (3) for ***M*** can be obtained by generalized inversion (Aki and Richards, 2002):

$$\boldsymbol{M}=(\mathbf{K}^{*}\mathbf{K})^{-1}\mathbf{K}^{*}\mathbf{U}^{(0)} \qquad (4)$$

where the star denotes complex conjugation and transposition.

Thus, since all the six independent components of moment tensor **M** contribute to the waveforms $\mathbf{U}^{(0)}$ in the over-determined system of Eq. 3 the inversion scheme of Eq. 4 should enable, at least theoretically, to obtain a unique solution for each of them. Within the limitations of current source presentation and path effects modeling, the solution is exact and convergence is reached after a single iteration. The inverse problem in this case consists in determining the parameters of the point source under condition that the source location and origin time is known, as well as distribution of velocities of seismic waves between the source and the station.

3. Application

In this section, the efficiency of the proposed inversion method is tested by applying it to an earthquake from the Kyoto region of Japan. For the purpose of this research, we selected a weak crustal earthquake that occurred on May 11, 2021, on Kyoto-Osaka border (06:08:44.03 UTC, 34.836°N, 135.616°E, Mw3.3, depth 12.49 km). Three-component waveforms observed by CEORKA (Committee of Earthquake Observation and Research in the Kansai Area) seismic stations are shown in Fig. 1. The seismic moment tensor of the earthquake, shown in Fig. 2, was determined by specialists of the Japanese National Research Institute for Earth Science and Disaster Resilience (NIED) by inverting the whole observed waveforms from three broadband stations F-net (Aoi et al., 2020; NIED, 2019). To test our method, we consider two cases: using records at four stations and at eleven stations.

1D crustal model used in all the inversions of waveforms is listed in Table 1 (Hallo et al. 2019). The source is located at a depth of 12.49 km. Note that the four stations: DIG, NRO, OCU, and MNZ are located in different quadrants around the epicenter of the earthquake. In the case of using eleven stations, to the above four stations are added records at seven more stations: DIG, NRO, OCU, MNZ, TOY, HSD, KM2, KTR, IHS, SMA, and TDO. The duration of direct waves at a station is estimated visually from the records, and accounting for the delays of reflection-conversion phases at the station corresponding to the model at a respective epicentral distance and source depth.

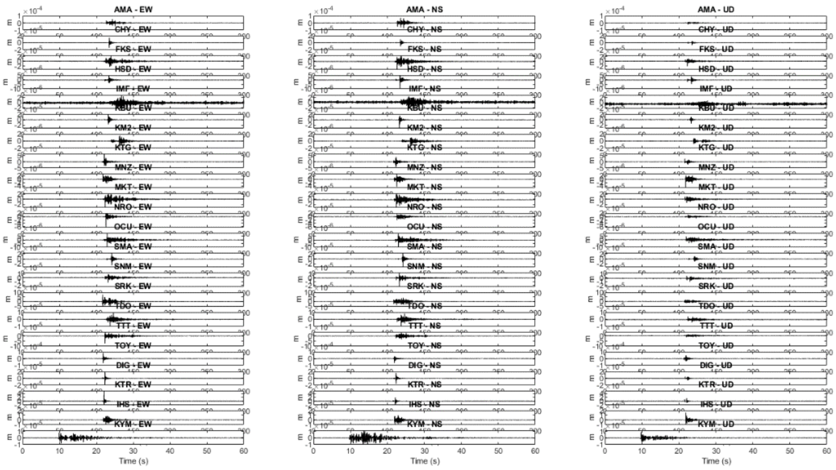


Figure 1. Records (in displacements) of the Kyoto-Osaka earthquake of May 11, 2021 (data from CEORKA, http://www.ceorka.org/).

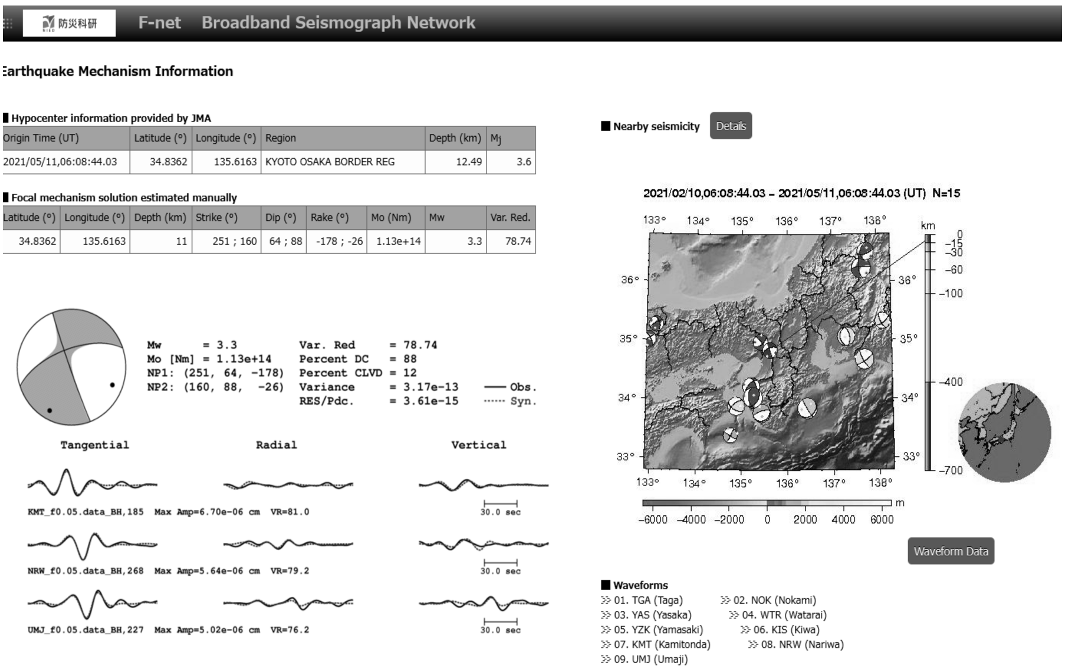
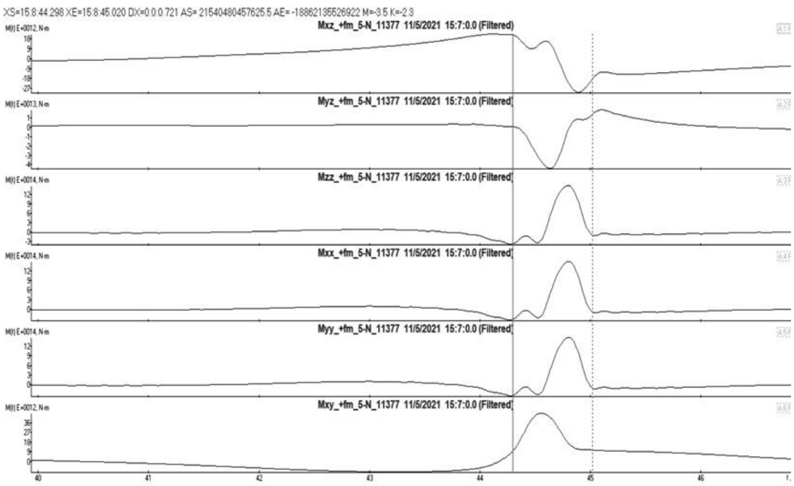


Figure 2. Focal mechanism of the Kyoto-Osaka earthquake obtained by the seismic moment tensor inversion by NIED F-net (https://www.fnet.bosai.go.jp/event/tdmt.php?_id=20210511060700&LANG=en)

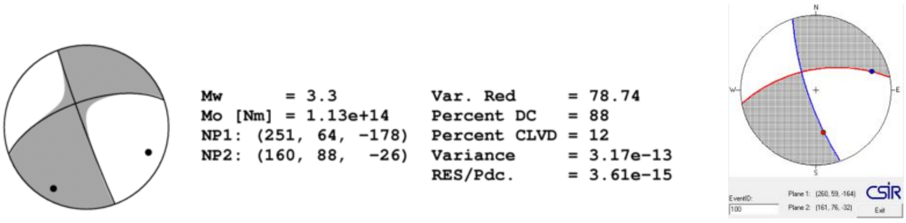
Table 1. 1D crustal model used in the inversion of waveforms.

Top depth of layers, km	Velocity V_p, km/s	Velocity V_s, km/s	*Density* ρ, g/cm^3
0.00	4.833	2.697	2.562
1.75	5.858	3.438	2.710
3.50	6.300	3.733	2.784
5.50	6.400	3.800	2.800
10.855	6.759	4.028	2.935
15.905	7.492	4.492	3.197

The seismic moment tensor was determined using our method by inverting the waveforms corresponding only to direct waves recorded at four stations: DIG, NRO, OCU, and MNZ (Fig.3, a): *Mxx*=0.52e14 Nm, *Mxy*=0.73e14 Nm, *Mxz*=0.04e14 Nm, *Myy*=-0.27e14 Nm, *Myz*=-0.51e14 Nm, *Mzz*=-0.24e14 Nm, and eleven stations: DIG, NRO, OCU, MNZ, TOY, HSD, KM2, KTR, IHS, SMA, and TDO (Fig.4, a): *Mxx*=0.40e14 Nm, *Mxy*=0.14e14 Nm, *Mxz*=-0.69e14 Nm, *Myy*=-0.52e14 Nm, *Myz*=0.53e14 Nm, *Mzz*=0.12e14 Nm of the local strong-motion seismological observation network CEORKA.

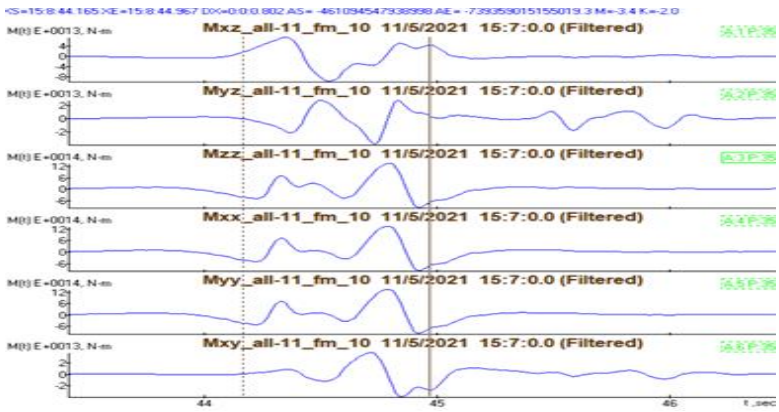


a)

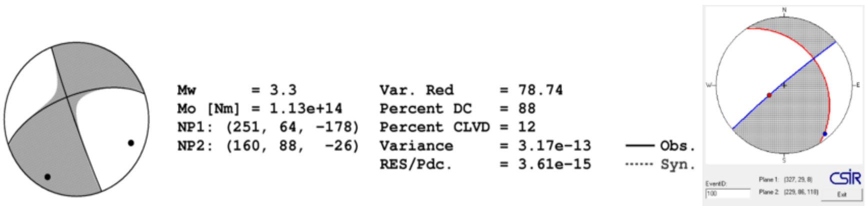


b)

Figure 3.
a) seismic moment tensor obtained for the 11 May 2021 earthquake by inverting the waveforms of only direct P- and S-waves at four stations (DIG, NRO, OCU, MNZ);
b) focal mechanism obtained by NIED F-net (left), and version of the focal mechanism solution corresponding to the seismic tensor a) (right).



a)



b)

Figure 4.
a) seismic moment tensor obtained by inverting the waveforms of only direct P- and S-waves at eleven stations (DIG, NRO, OCU, MNZ, TOY, HSD, KM2, KTR, IHS, SMA, and TDO);
b) focal mechanism obtained by NIED F-net (left), and version of the focal mechanism solution corresponding to the seismic tensor a) (right).

From the comparative analysis of the results shown in Figures 3 (b) and 4 (b), we can conclude that the use for inversion of only direct waves registered at a limited number of stations located around the epicenter of the earthquake gives satisfactory results for the focal mechanism. To conclude, the seismic moment tensor inversion is important but non-trivial task and the number of observation points influences the precision of the solution. Still, when using appropriate advance techniques and carefull data quality check, it is possible to infer the focal solution from a low number of observational points.

4. Discussion and Conclusions

In the paper, a method is presented for moment tensor inversion of only direct P- and S-waveforms registered at only several stations. It is theoretically shown that the displacements at every single point on the upper surface of horizontally layered and perfectly elastic half-space generated by the point source represented by time-varying symmetric moment tensor depend on all of its six components.

The method is based on an inversion approach described in (Malytskyy, 2010, 2016) where a version of matrix method has been developed for calculation of direct waves in horizontally layered half-space from the point source represented by its moment tensor. Using the method presented in the current paper focal mechanisms of the M3.6 earthquake on 11 May 2021 (06:08:44.03 UTC, 34.836°N, 135.616°E, depth 12.49 km) in the Kyoto- Osaka border region was retrieved. Choosing to invert only the direct waves, calculated by matrix method, instead of the full wave field, enables us to reduce the effects of the half-space model inaccuracy, reflected and converted phases being much more distorted by it. The assumption of horizontally layered half-space, as well as the distribution of seismic velocities in it, may however turn out grossly incorrect in fact. Combined with inaccurate knowledge of source location and origin time, as well as with a number of the other uncertainties, such as introduced by seismic noise in the observed seismograms etc., it may almost completely obscure the source imprint in the seismograms, and especially in those originating, as it is, from only limited number of seismic stations, turn the moment inversion ill-defined and lead to an intractable solution. This is why some independent controls for the inversion results are needed. However, such studies are very necessary for regions with low seismicity, where records from only a limited number of stations can be used to determine the parameters of the earthquake source, which is important for seismology and geophysics in general. Continuation of such research is planned in future works of the authors.

Acknowledgements

This research is a result of research visit in Japan supported by the Disaster Prevention Research Institute (DPRI) of Kyoto University (project number 2022L-01). In this study, we used earthquake waveforms recorded by seismic stations operated by the CEORKA (Committee of Earthquake Observation and Research in the Kansai Area, http://www.ceorka.org/). The authors are grateful to Dr. Miroslav Hallo for the presented records at the stations and useful advice during the discussion of the results.

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Received September 23, 2024, accepted November 5, 2024