

# Preliminary Reassessment Results of the Recent Vertical Movements of the Earth's Crust in Bulgaria

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**Abstract.** Recent vertical crustal movements in Bulgaria have been mapped using adjusted levelling data from three epochs, yet results from different research teams are inconsistent and often contradict known tectonic fault structures. To provide an independent evaluation, we applied a new estimation approach based on data from Bulgaria's Second (1953–1957) and Third (1975–1980) levelling campaigns. Both networks were processed through 3<sup>rd</sup> independent adjustments, selecting line elevations that minimised loop misclosures. In addition, Inverse Distance Weighting (IDW) with a power parameter of 6 was applied. This procedure reduced benchmark height standard errors to below  $\pm 5.3$  mm and vertical velocity errors to  $\pm 0.30$  mm/year. The resulting high-accuracy map reveals a strong correlation between tectonic boundaries and major earthquake epicentres.

**Keywords:** Probability Theory, Recent Movements of Earth's Crust, Geometric Levelling, Adjustment Algorithms, Earthquake Hazards

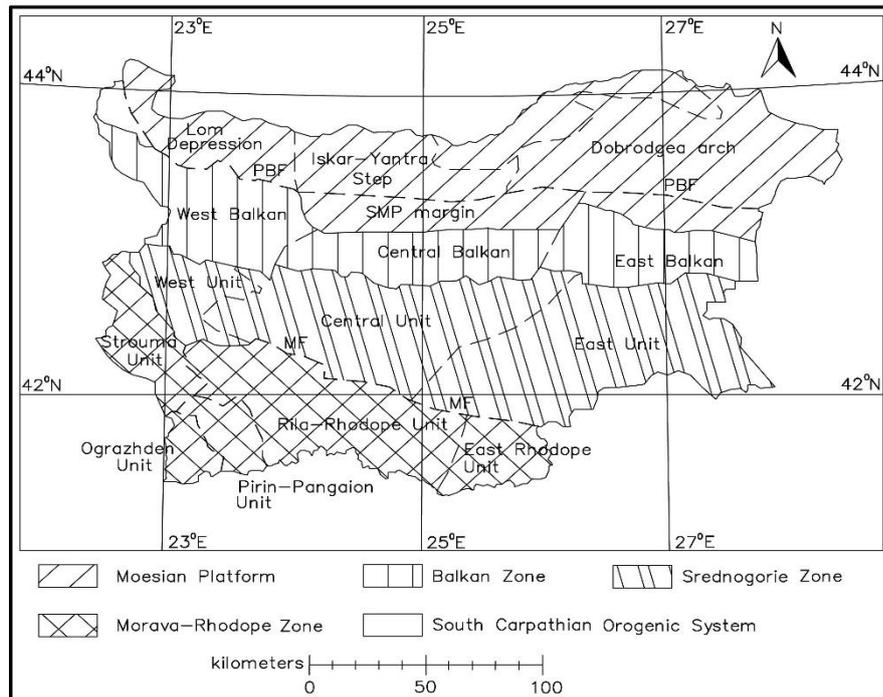
## 1. Introduction

### 1.1. Tectonic scheme of Bulgaria

Bulgaria lies within two major tectonic zones: the northern part of the Alpine thrust belt in the Balkans and its foreland, the Moesian Platform. Bulgarian geologists widely recognise these units, but debates persist over how to divide them further, interpret their structures, and understand their geodynamic history.

Following the widespread acceptance of plate tectonic theory around forty years ago, a variety of competing models have been proposed to interpret Bulgaria's tectonic structure and geological evolution. Most of them are built on the idea that the Alpine development of the Balkans was shaped by ongoing subduction and multiple continental collisions along Eurasia's southern margin. While these models differ in how they define tectonic units, they generally agree on one point: the Alpine thrust belt is a patchwork of both local and exotic continental fragments that accreted to Eurasia during the closure of the Tethys Ocean.

Despite this long-standing interest, no up-to-date tectonic map of Bulgaria has been published. Developing a reliable tectonic model requires a comprehensive analysis of geological data, including lithology, stratigraphy, structure, paleontology, volcanism, plutonism, metamorphism, geochronology, and geophysics. This is the approach the authors (Dabovski, C., Boyanov, I., Khrishev, K., Nikolov, T., Sapounov, I., Yanev, Y., and Zagorchev, I., 2002, Figure 1) have taken in creating a new model for the Alpine structure and evolution of Bulgaria, shown in Figure 1.

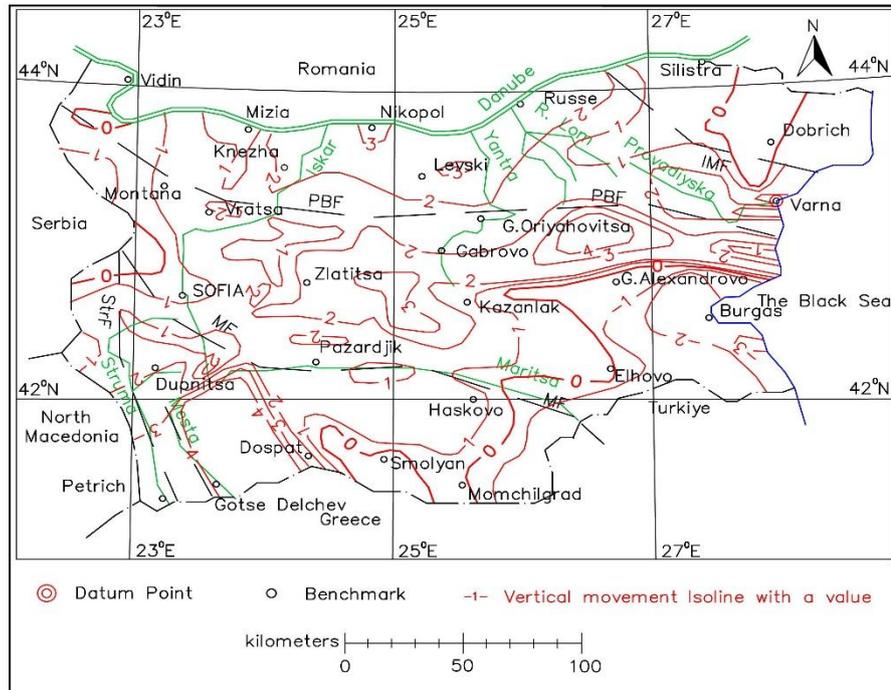


**Fig. 1.** Tectonic Scheme of Bulgaria. (Based on Dabovski, C., Boyanov, I., Khrishev, K., Nikolov, T., Sapounov, I., Yanev, Y. and Zagorchev, I., 2002, Figure 1)

In this model, tectonic units are understood as rock bodies that are confined dominantly between fault structures. Each unit is characterised by a specific rock assemblage (specific lithology, stratigraphy, magmatism, metamorphism) and deformational events (age, resulting structures), recording a geological evolution different from that of the adjacent units (Dabovski, C., Boyanov, I., Khrishev, K., Nikolov, T., Sapounov, I., Yanev, Y. and Zagorchev, I., 2002, Figure 1).

## 1.2. Kanev and Mladenovski's map from 1969

One of the most popular maps of the recent vertical movements and their speeds in the territory of Bulgaria was created by Kanev and Mladenovski (Kanev and Mladenovski, 1969). The map, shown in Figure 2, is based on the results from the First (1920-1930) and the Second (1953-1957) Levelling campaigns in our country.



**Fig. 2.** Kanev and Mladenovski's map from 1969. (Kanev, D. and Mladenovski, M., 1969, Figure 2)

According to Figure 2, almost the whole territory of Bulgaria is rising. The magnitude of the uplifts is mainly 1-2 mm/y. The supreme uplift velocities, with values exceeding 4 mm/y, are in the Pirin mountain and the Kotel-Omurtag part of the Balkan /Stara Planina/. The sinking areas are the South-Middle Rhodopes and the Strandja-Sakar mountains, where the drop speeds of the Earth's crust are up to -3 mm/y. The standard errors of the estimated vertical speeds used weights and additional details regarding the adjustment of the levelling networks are not given in the publication (Kanev and Mladenovski, 1969).

### 1.3. Spiridonov and Georgiev's map from 2003

Another assessment of recent vertical crustal movements in Bulgaria is provided by Spiridonov and Georgiev (2003). Their results indicate subsidence within the Lom Depression at a rate of approximately -2.5 mm/year, whereas the central Moesian Plain exhibits uplift of about 1 mm/year. The Western Forebalkan and much of the Balkan region are subsiding at approximately -1 mm/year, whereas the Central Balkan is rising at 2 mm/year. Uplift rates for the Rila, Pirin, Rhodopes, and Sakar mountains are 2 mm/year, 4 mm/year, 3.5 mm/year, and 1.5 mm/year, respectively. The study does not provide standard errors for these values. The authors themselves note a lack of geomorphological consistency between their results and the pattern of known active faults in Bulgaria. Notably, the South-Moesian fault [7, 8] is absent from their findings.

#### 1.4. Belyashki's map from 2012

A different picture of the tectonic motions in the territory of Bulgaria is presented by Belyashki (Belyashki, T., 2012, Figure 3). Using the first and the second order levelling data from the First and the Third Levelling campaign in Bulgaria, Belyashki created the map presented in Figure 3.

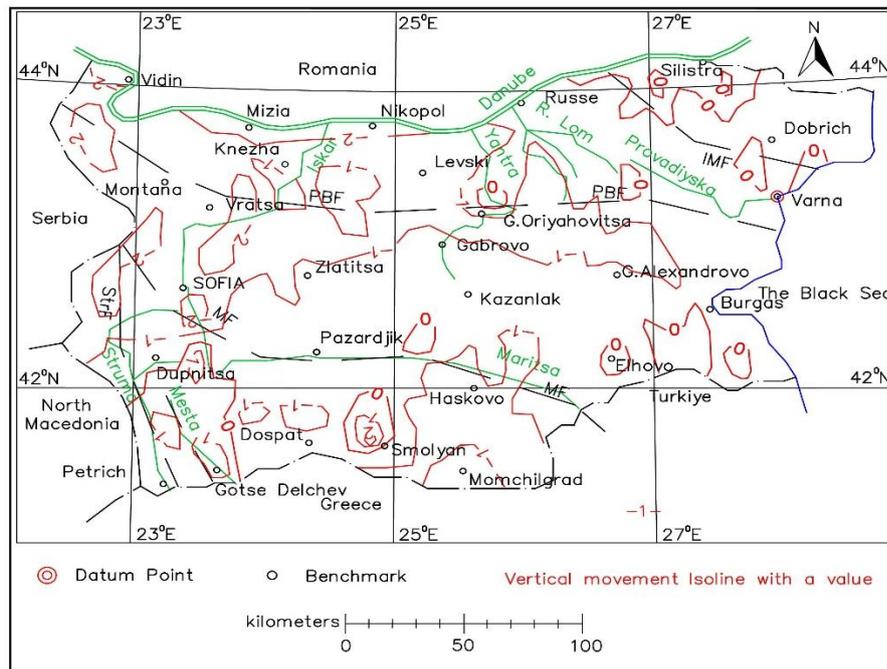


Fig. 3. Belyashki's map from 2012. (Belyashki, T., 2012, Figure 3)

The Belyashki's approach was to estimate relative speeds between line terminal benchmarks on the first stage. In the second stage, he used relative speeds to close loops, which were adjusted using weights (1).

$$w = 100 \cdot \Delta t^2 / [40^2 \cdot L \cdot (m_I^2 + m_{III}^2)] \quad (1)$$

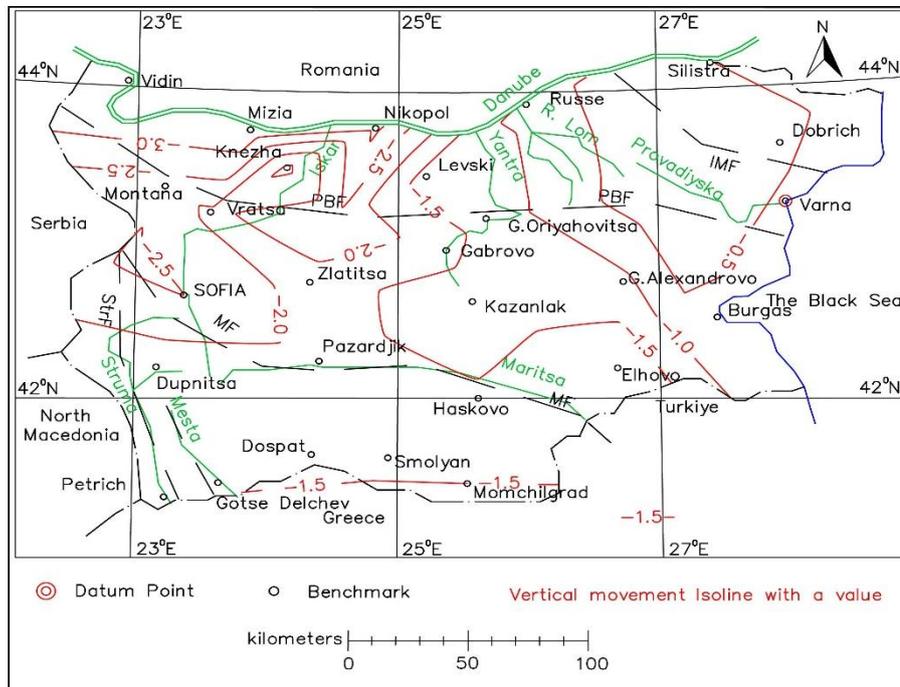
In equation (1),  $L$  is the length of the levelling line, and  $m_I$  and  $m_{III}$  are the a posteriori accuracy of the First and the Third Levelling Campaigns.

According to Figure 3, the velocities are predominantly in the range from 0 to -2 mm/y. The velocities between -2 mm/year and -3.5 mm/year are in the Lom depression and the Western Forebalkan. The mountains Pirin, Rila, and Central Rhodopes are rising by 1-2 mm/y. A similar rise is also detected in the Lodogorie-Dobrudzha Swell and the Strandzha

mountain. The standard errors of the obtained velocities vary from 0.1 mm/year in Eastern Bulgaria to 1 mm/year in Western Bulgaria, with an average of 0.30 mm/year.

### 1.5. Gospodinov's team map from 2022

The most recent map of vertical crustal movement in Bulgaria was published by Gospodinov et al. (2022). That study presented three different representations of vertical tectonic motion, each varying in content, conclusions, and accuracy. The most reliable of them, based on the reported standard deviations of benchmark vertical speeds, is the map derived from levelling data collected during Bulgaria's First and Third national levelling campaigns. This version is shown in Figure 4 below.



**Fig. 4.** Gospodinov's team map from 2022. (Gospodinov et al., 2022, Table 4, Column 7)

The applied approach is the simultaneous adjustment of networks from both epochs. The used weights are in the form (2).

$$w = 1/[\mu^2 \cdot L + \sigma^2 \cdot L^2] \quad (2)$$

In equation (2),  $L$  is the length of the levelling line, and  $\mu$  and  $\sigma$  are the accidental and systematic errors estimated by the concrete line.

Contrary to Kanev and Mladenovski, Gospodinov's team (Gospodinov et al., 2022, Table 4, Column 7) estimates that the whole territory of Bulgaria is sinking. According to the authors, the values of the velocities are mainly in the range  $-1 \div -2$  mm/y. The maximum fall of  $-3.4$  mm/year is in the Lom depression, located in the southwest part of

the Moesian Platform. The standard errors of the determined velocities are 0.70 mm/year on average.

## 1.6. Research objectives

The common issues in the vertical velocity estimates from the studies above are:

- They used the mean of paired line elevation measurements during levelling adjustments - a major source of systematic error in levelling data processing (Cvetkov, 2022, 2024a, 2024b).
- Adjustments were made without minimising the standard errors of the adjusted benchmark heights (Cvetkov and Gospodinov, 2023).
- Adjustments used weights inversely proportional to the line weights. According to newly revealed facts, there is no clear and statistically proven relationship between the levelling lines and line elevation accuracy (Cvetkov, 2022, 2023, 2024c).
- Using the same data, these studies produced inconsistent outcomes that don't align with Bulgaria's known active tectonic faults (Dabovski et al., 2002; Spiridonov and Georgiev, 2003; Zagorchev, 2009; Stanciu and Ioane, 2023).

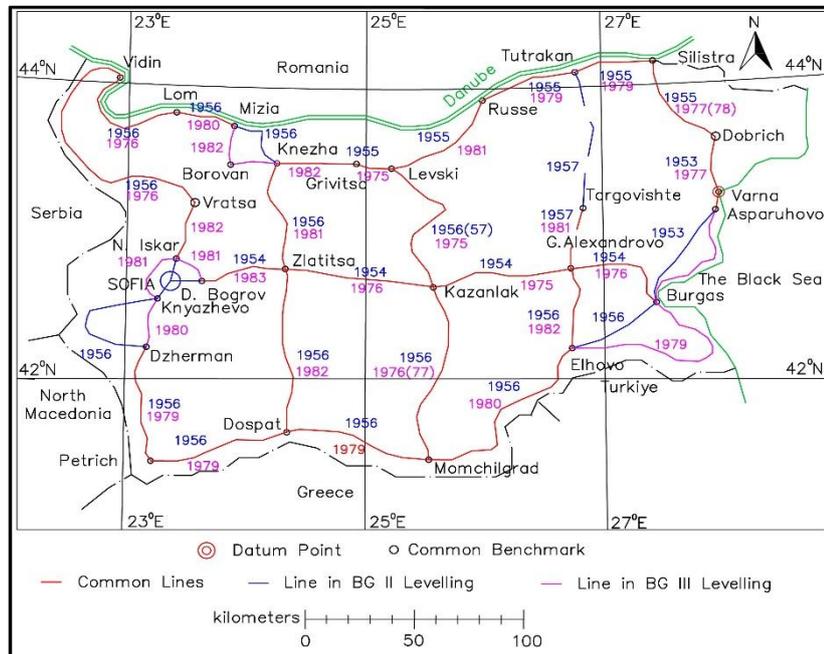
So, we define our main goal as a reassessment of the recent vertical crustal movements in Bulgaria using updated approaches grounded in recent developments in probability theory (Cvetkov, 2024a, 2024b, 2024c), modern geometric levelling adjustment methods (Cvetkov, 2024c; Cvetkov and Gospodinov, 2023), and sophisticated tests for analysing benchmark vertical displacements and their speeds (Caspary, 2000; Hamza et al., 2020; Niemeier, 1981; Savšek-Safić et al., 2006; Setan and Singh, 2001; among others).

## 2. Data and its processing

### 2.1. Used data

Appendices B and C contain the levelling data for the Second Levelling (1953–1957) and the Third Levelling of Bulgaria (1975–1984), respectively, used in this study. Figure 5 shows the configurations of the networks, along with the corresponding measurement years. The total lengths are 3,127.47 km for the Second Network and 3,568.33 km for the Third. It must be remarked that the nodal benchmarks in both networks were selected to meet the following criteria:

- Common for both networks.
- Mounted on massive public buildings such as railway stations, post offices, administrative buildings, churches, etc., or on rocks.
- All common benchmarks were estimated as stable by checking their stability against at least three neighbouring benchmarks.



**Fig. 5.** Configuration of the Second Levelling Network /1953-1957/ and that part of the Third Levelling Network /1975-1984/ of Bulgaria, used in this analysis.

## 2.2. Data processing

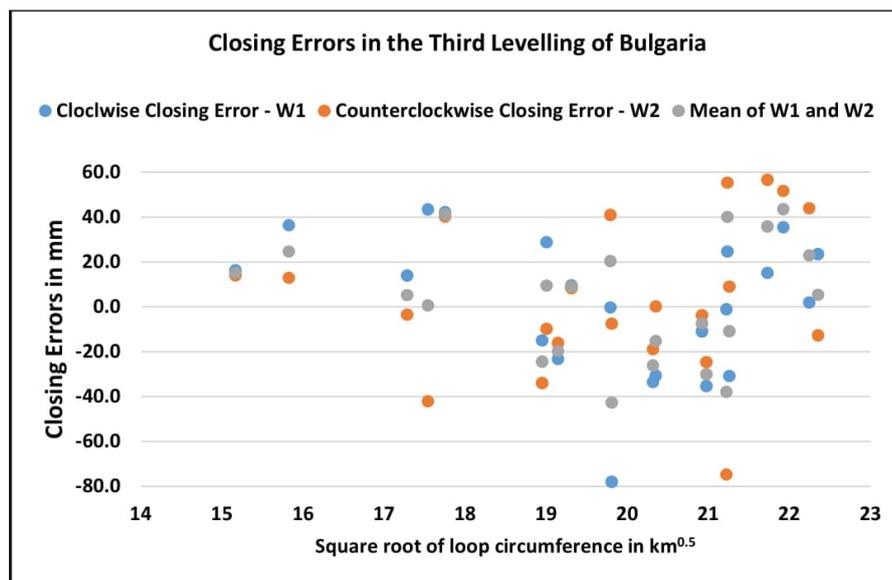
Figure 5 shows that shared routes between the two levelling epochs predominate over routes unique to each epoch. This overlap improves the reliability and objectivity of the estimated vertical displacements and velocities (Caspary, 2000; Hamza et al., 2020; Niemeier, 1981; Savšek-Safić et al., 2006; Setan and Singh, 2001). Still, there are notable differences in the network configurations. The Second Levelling was primarily conducted between 1955 and 1956, while the relevant portion of the Third Levelling occurred between 1975 and 1983. During that time, some benchmarks shifted, resulting in a reduction in the final network accuracy.

The equipment and methods also varied between epochs. The Second Levelling used spirit levels - primarily Zeiss III and Ni004. The Third Levelling relied on automatic levels with compensators, using both first-order Ni002 and, after 1980, second-order Ni007 instruments. Measurement procedures also differed: the maximum sight length was 50 m in the Second Levelling, but reduced to 40 m in the Third.

Given these variations, a more advanced and refined analysis of the levelling data is justified (Caspary, 2000; Hamza et al., 2020; Niemeier, 1981; Savšek-Safić et al., 2006; Setan and Singh, 2001). The processing algorithm used is described below.

### 2.2.1. Selecting those observations in levelling lines that minimise the loop closing errors by a separate 3<sup>n</sup> adjustments of each epoch network

The major drawback of the classical adjustment method for precise levelling networks is the use of the arithmetic mean of the two measured height differences between the endpoints of a levelling line. Of course, suppose we assume that both measurements of each height difference follow the same distribution with a standard deviation of  $\sigma$ . In that case, the arithmetic mean of the two measurements will have a standard deviation of  $0.707\sigma$ . However, this fact does not mean that the true error of the arithmetic mean of the two measurements is smaller than the true error of either individual measurement. In reality, the probability that the true error of the arithmetic mean of two random observations is smaller than the true errors of both observations converge to 33% if the distribution of the observations is uniform. In the case of normally distributed observations, this probability is even lower, under 30% (Cvetkov, 2022, 2024a, 2024b). Figure 6 illustrates this point using the closing errors in the loops from Bulgaria's Third Levelling. The procedure of 3<sup>n</sup> adjustments is widely discussed in (Cvetkov, 2024c, 2024d). Appendix A presents the pseudocode for the 3<sup>n</sup> selection procedure, which was implemented in a C# language program to perform the computations in this research.



**Fig. 6.** Loop closing errors in the Third Levelling Network /1975-1984/ of Bulgaria.

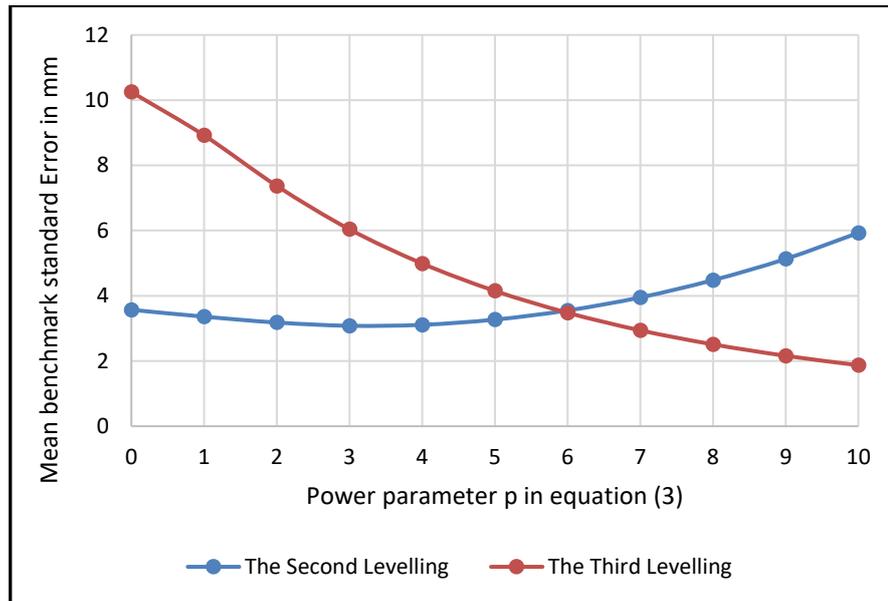
Figure 6 shows that in over 70% of cases, the mean of observations W1 and W2 has a larger true error than either W1 or W2 individually. This suggests that using the observation with the smallest true error, rather than their mean, is a more robust and reliable method, especially in the presence of gross errors, outliers, and systematic errors, which are common in high-precision geometric levelling.

### 2.2.2. Minimising the outlier effects and network configuration influence by applying IDW to the networks

After selecting the most reliable observations, we carried out the final adjustments for both the Second and Third Levelling Networks. To minimize the impact of network configuration on the results, we employed iterative reweighting using IDW. The weights were defined according to equation (3).

$$w = (150(km)/L(km))^p \quad (3)$$

In equation (3),  $L$  denotes the length of levelling lines and  $p$  the power parameter. This approach further reduces the standard errors of the adjusted benchmark heights, resulting in improved accuracy of the vertical displacement estimates (Cvetkov and Gospodinov, 2023). As illustrated in Figure 7, the intersection of the summed benchmark standard errors, defined as the mean standard errors derived from the benchmark standard error samples of the Second and Third Bulgarian Levelling, occurs at a power parameter of approximately  $p \approx 6$ . Consequently, a value of  $p = 6$  was adopted in this study. This approach is essential not only for achieving closer alignment of the central tendencies of the benchmark standard error samples but also for ensuring greater sample homogeneity, which is critically important in displacement analysis. The homogeneity of the benchmark standard errors obtained in both measurement epochs is examined in Subsection 2.2.3.



**Fig.7.** Mean values of benchmark's standard errors regarding the power parameter  $p$  in equation (3).

Appendix D presents the adjusted benchmark heights for both measurement epochs together with the derived benchmark vertical velocities (mm/year), calculated using IDW

power parameters  $p = \{4, 5, 6, 7, 8\}$ . The results show that selecting  $p$  within this range has no significant effect on the computed benchmark velocities. Consequently, the delineation of the principal tectonic zones remains stable under small variations of the IDW power parameter.

### 2.2.3. Test the a posteriori variance factors of both epochs for their compatibility

When identifying displacements, network reliability and sensitivity are critical. Therefore, it's essential to detect any hidden gross errors and ensure uniform accuracy across both networks. These conditions can be evaluated by comparing the a posteriori variance from both adjustments (Caspary, 2000; Hamza et al., 2020; Niemeier, 1981; Savšek-Safić et al., 2006; Setan and Singh, 2001). The hypotheses for this test are the null hypothesis ( $H_0$ ) and the alternative hypothesis ( $H_a$ ).

$$H_0: \sigma_1^2 = \sigma_2^2 \quad (4)$$

$$H_A: \sigma_1^2 \neq \sigma_2^2 \quad (5)$$

Verification of the null hypothesis can be done by the test statistic T (6):

$$T = \frac{\sigma_1^2}{\sigma_2^2} < F_{1-\alpha, f_1, f_2} \quad (6)$$

$$f_i = n_i - k_i - d_i \quad (7)$$

$\alpha$ -risk level,

$f_i$  - degree of freedom of a certain epoch,

$n_i$  - number of observations of a certain epoch,

$k_i$  - number of unknowns of a certain epoch,

$d_i$  - datum defect of a certain epoch.

In our final adjustments, we used 20 lines defined by the following benchmarks: Varna, Burgas, Gorno Alexandrovo, Elhovo, Tutrakan, Kazanlak, Levski, Momchilgrad, Dospat, Zlatitsa, Knezha, Vidin, Petrich, and Sofia. In the case of the Third Levelling, the nodal benchmark in Sofia was selected in Novi Iskar. The century benchmark 28 in Varna was adopted as the datum point, with an estimated vertical velocity of  $\pm 0.00$  mm/year (Belyashki, 2012). According to a FIG report (Sacher et al., 2004), the height of benchmark 28 in Varna had changed by only 2 mm over 25 years (from 1958 to 1983). In addition, this benchmark was selected to maintain consistency with previous network adjustments (Gospodinov et al., 2022; Belyashki, 2012; Kanev and Mladenovski, 1969), which report its long-term stability. Thus, we have that  $n_1=n_2=20$ ,  $k_1=k_2=13$ ,  $d_1=d_2=0$ , or  $f_1=f_2=7$ .

If the null hypothesis (4) is not rejected, then the accuracy of both networks is compatible. There are no gross errors in observations. There is a significant similarity in the network configurations. Thus, the assessment of vertical displacements will produce unbiased and reliable estimates.

If the null hypothesis (4) is rejected, further analysis must be halted. At this point, a deeper investigation is required - reassessing the applied weights, identifying undetected

outliers, and examining differences in network configuration (Casparly, 2000; Niemeier, 1981; Savšek-Safić et al., 2006).

#### 2.2.4. Estimation of the speed of vertical displacements and its accuracy

Before estimating the rate of vertical crustal displacements and their annual speeds, we first need to determine the central epochs of the two levelling campaigns - specifically, the period  $\Delta t$  between them. This is calculated as a weighted average of the measurement years, using the total length of levelling lines recorded each year as weights. For the Second Levelling Network, the weighted average year is 1955.22; for the Third, it is 1978.44. Therefore, the period is  $\Delta t = 1978.44 - 1955.22 = 23.22$  years.

Suppose that  $\sigma_{H_t}^2$  and  $\sigma_{H_{t+\Delta t}}^2$  are the variances of a random nodal benchmark in the networks of the Second and Third Levelling, and  $H_t$  and  $H_{t+\Delta t}$  are the adjusted heights of this benchmark in the epochs 1955.22 and 1978.44, the displacement of this benchmark and the accuracy of the displacement can be given by equations (8) and (9).

$$\Delta = H_{t+\Delta t} - H_t \quad (8)$$

$$\sigma_{\Delta}^2 = \sigma_{H_t}^2 + \sigma_{H_{t+\Delta t}}^2 \quad (9)$$

Assuming that the errors of observations are distributed normally  $\varepsilon \sim N(0, \sigma^2)$ , then the adjustment parameters  $dH_t$  and  $dH_{t+\Delta t}$  would also be distributed normally. Since the displacements  $\Delta$  are differences between  $dH_t$  and  $dH_{t+\Delta t}$ , under the assumption of an equal approximate initial height  $dH^0$  of the random benchmark in both epoch adjustments, the displacements  $\Delta$  would also be normally distributed. Thus, the ratio (10) would follow the standard Normal distribution.

$$T = \frac{\Delta}{\sigma_{\Delta}} \sim N(0, 1) \quad (10)$$

If  $T \geq N(0, 1, \alpha)$ , then we can conclude at a significance level higher than  $1-\alpha$  that the quantity  $\Delta$  is a real displacement, but not a measurement error. More sophisticated estimation of the statistic  $T$  through simulations is also possible (Savšek-Safić et al., 2006).

Finally, the velocity of a random nodal benchmark common to both epochs can be computed using equation (11).

$$V = \frac{\Delta}{\Delta t} = \frac{\Delta}{23.22} \quad (11)$$

The accuracy of the speeds  $V$  is directly given by (12) (Kääriäinen, 1966, pp. 59).

$$\sigma_V = \frac{\sigma_{\Delta}}{\Delta t} = \frac{\sigma_{\Delta}}{23.22} \quad (12)$$

Thus, statistic (10) is a quite valid estimator for assessing the significance of the speeds in equation (11).

$$T = \frac{V}{\sigma_V} = \frac{\Delta/\Delta t}{\sigma_{\Delta}/\Delta t} = \frac{\Delta}{\sigma_{\Delta}} \sim N(0, 1) \quad (13)$$

### 3. Results

Table 1 shows the adjusted heights of common nodal benchmarks for epochs 1955.22 and 1978.44, along with their standard deviations. Table 4 presents the F-Test results comparing the a posteriori accuracy from both adjustments. Table 3 lists the estimated benchmark displacements, their velocities, associated uncertainties, and statistical significance. Figure 8 maps the recent ground movement speeds across Bulgaria, based on this analysis.

**Table 1.** Adjustment results.

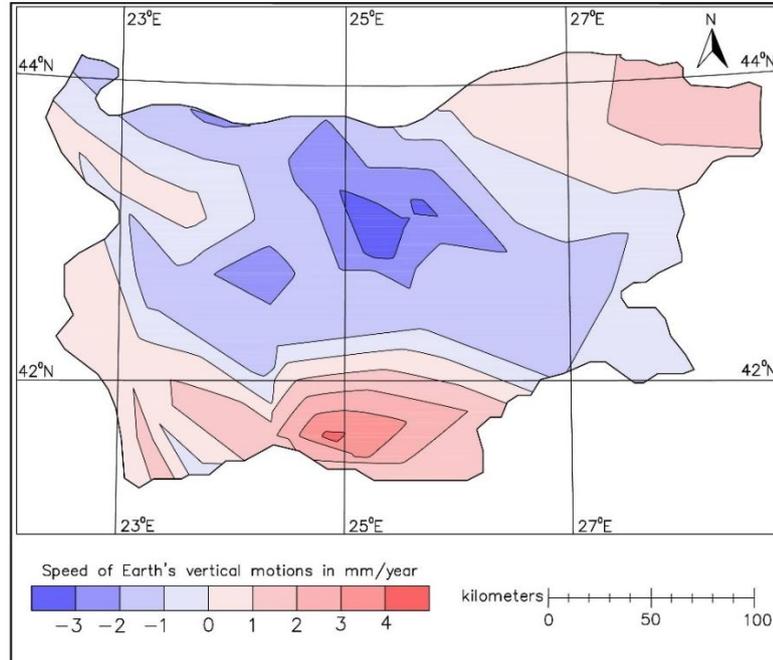
Benchmark	1955.22		1978.44	
	H m	$\sigma_H$ (mm)	H m	$\sigma_H$ (mm)
Burgas	82.46760	2.06	82.45194	2.25
G. Alexandrovo	154.12936	2.09	154.09012	2.26
Elhovo	128.47842	2.09	128.43560	2.27
Tutrakan	35.20201	4.49	35.22153	4.61
Kazanlak	376.47839	2.60	376.43639	2.79
Levski	70.47135	3.19	70.42584	3.24
Momchilgrad	250.81428	4.42	250.85406	4.35
Dospat	1196.32312	4.57	1196.36630	4.35
Zlatitsa	674.71126	3.01	674.64775	3.02
Knezha	161.39332	3.14	161.35912	3.18
Vidin	42.06641	4.69	42.03004	5.30
Sofia (N. Iskar)	515.92643	3.04	515.87861	3.07
Petrich	110.40077	5.03	110.39552	4.54

**Table 2.** Test of the a posteriori variance factors for both epochs for compatibility, performed at the significance level  $\alpha = 0.05$ .

Statistic	Value
$\sigma_{1955.22}$	2.8166
$\sigma_{1978.44}$	2.2354
Fobs	1.5876
F(7,7,0.05)	3.7870
p-Value	0.2784

**Table 3.** Estimated benchmark displacements, their speed, accuracy, and statistical significance.

<b>Benchmark</b>	<b><math>\Delta</math></b> <b>(mm)</b>	<b>V</b> <b>(mm/year)</b>	<b><math>\sigma_v</math></b> <b>(mm/year)</b>	<b>Tobs.</b>	<b>N(0, 1, T<sub>obs</sub>, two-tail)</b>
Burgas	-15.65	-0.67	0.13	5.13	<b>2.862E-07</b>
G. Alexandrovo	-39.25	-1.69	0.13	12.75	<b>0.000E+00</b>
Elhovo	-42.82	-1.84	0.13	13.86	<b>0.000E+00</b>
Tutrakan	19.52	0.84	0.28	3.03	<b>2.406E-03</b>
Kazanlak	-42.00	-1.81	0.16	11.01	<b>0.000E+00</b>
Levski	-45.51	-1.96	0.20	10.00	<b>0.000E+00</b>
Momchilgrad	39.78	1.71	0.27	6.41	<b>1.433E-10</b>
Dospat	43.18	1.86	0.27	6.85	<b>7.556E-12</b>
Zlatitsa	-63.52	-2.74	0.18	14.91	<b>0.000E+00</b>
Knezha	-34.20	-1.47	0.19	7.65	<b>1.954E-14</b>
Vidin	-36.37	-1.57	0.30	5.14	<b>2.749E-07</b>
Sofia (N. Iskar)	-47.82	-2.06	0.19	11.07	<b>0.000E+00</b>
Petrich	-5.25	-0.23	0.29	0.77	4.385E-01

**Fig.8.** The new map of the recent vertical movements of the Earth's crust in Bulgaria.

## 4. Discussion

Table 1 shows that the standard deviations of the adjusted nodal benchmark heights are within 5 mm for both network epochs, indicating strong consistency. The adjusted benchmark heights are highly similar between epochs, a fact confirmed statistically by the compatibility test of a posteriori variance factors in Table 3. These results show that both networks have equal accuracy, meaning the estimation of vertical displacements won't be compromised by poor weighting models, outliers, or major differences in network design.

As a result, the displacement estimates, along with their velocities and accuracy figures in Table 2, are statistically valid. Figure 8, based on Table 2, illustrates that Bulgaria can be divided into five regions according to the direction and magnitude of vertical crustal movement. The first one is the Eastern part of the Lodogorie-Dobrudzha Swell. This part of Bulgaria is rising by 1 mm/year. The boundary, that is to say, the velocity zero line, starts from Varna Bay, passes along the rivers Pomoriyska, Beli Lom, and Rusenski Lom. Thus, this zero line is very close to the Intramoesian Fault sketched by Stanciu and Ioane (2023, Figure 2) and coincides with a group of active faults given in this sketch.

The second part includes the rest of the Moesian Platform (Lom Depression, Iskar-Yantra step, South Moesian Platform margin), the Forebalkan, Central and East Balkan, Srednogorie unit, and the Strandzha-Sakar massifs. This entire territory is sinking at a rate of 1-2 mm/year. The only exception is the area along the Yantra river between the towns of Gabrovo and Gorna Oriyahovitsa, where the velocities of sinking are up to -3.5 mm/year. This sinking is likely a logical result of the intensive seismic activities in this place (Stanciu and Ioane, 2023, Figure 11). Our results confirm the subsidence of the Lom depression at a rate of 1.5-2.5 mm/year.

The south boundary of the second part is the Maritsa fault. South of this line, which is an agglomerate of important neotectonic faults, lies the third tectonic part in Bulgaria, the Morava-Rhodope zone, which is rising by 1-2 mm/year. The supreme rise of +4.3mm/year we registered in the area of Smolyan town, which is close to the highest Rhodopes' peak – Golyam Perelik. Other rises more significant than 3 mm/year we yielded in Madan / 3.5 mm/year /, and Zlatograd / 3.2 mm/year /. Considering the Pirin Mountain, we found that the territory of Papas Chayr passage is rising by 1.7 mm/year. The area around Kroupnik town, known for its extensive seismic activity, is growing by 1.4 mm per year.

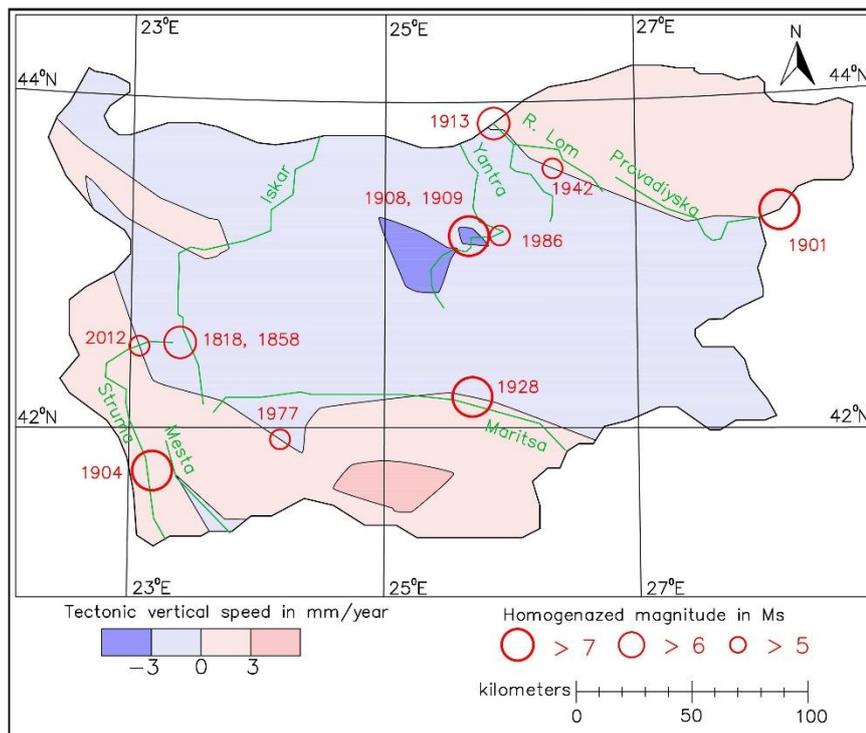
The only zone south of the Maritsa fault, where we registered sinking, is the area of the Mesta River fault. According to our calculations, the area around Gotse Delchev is sinking by 0.5 mm/year.

The last fifth zone is the West Balkan unit, located west of the Iskar River. The Forebalkan fault in the North surrounds this zone, the Iskar River fault in the East, and the boundary between the Forebalkan and the Balkan in the South. The registered rise between Belogradchik and the Prevala passage is approximately 0.8 mm/year, which is higher than the standard errors of the velocity multiplied by three.

In conclusion, the velocities of vertical crustal movements in Bulgaria generally range within  $\pm 3$  mm/year, except in the area surrounding the peak of Golyam Perelik in the Central Rhodopes, where they are over +4mm/year. These rates are comparable to those reported for Lithuania (Puziene et al., 2013). According to Puziene (2011), vertical movement velocities of this magnitude can significantly compromise the stability and accuracy of national reference systems. An analysis of selected lines in the Lithuanian levelling network demonstrated that vertical tectonic motions of similar rates reduce the accuracy of first-order levelling by approximately 67% over a period of ten years.

Complete degradation of the highest-order precise levelling network is expected within about 20 years, while second-order levelling networks may remain reliable for up to 30 years. Based on these findings, the highest-order Bulgarian levelling network should be remeasured at intervals of approximately 15–20 years to account for ongoing vertical tectonic activity in the region.

Another important aspect of the derived vertical tectonic velocities concerns the relationship between their signs and magnitudes and seismic hazard assessments. As discussed above, the boundaries between zones of positive and negative vertical velocities delineated on the new vertical tectonic map of Bulgaria closely follow well-known active faults. Figure 9 illustrates the spatial relationship between the epicentres of the strongest earthquakes in Bulgaria over the past two centuries and the boundaries separating zones of opposite vertical motion. Notably, the epicentres of major earthquakes, such as those in Kresna (1904, surface-wave magnitude  $M_s > 7$ ) (Bayliss and Burton, 2007; Dimitrov and Nakov, 2022), Plovdiv and Chirpan (1928,  $M_s > 7$ ) (Bayliss and Burton, 2007), and Shabla (1901,  $M_s = 7.2$ ) (Matova, 2000), are located along these boundaries. These zones coincide with Bulgaria's most significant active fault systems, including the Maritsa Fault (as depicted by Dabovski et al., 2002, Fig. 1), the Struma Fault, and the Intramoesian Fault (Stanciu and Ioane, 2023, Fig. 2).



**Fig. 9.** The largest earthquake in Bulgaria in the last two centuries.

As shown in Figure 9, the epicentres of the earthquakes in Gorna Oryahovitsa (1908 and 1909, with surface-wave magnitudes  $M > 7$ ) (Bayliss and Burton, 2007) and in

Stazhitsa (1986,  $M_s \approx 5.6$ ) are situated within the zone exhibiting the highest subsidence in Bulgaria. The epicentres of the earthquakes in Razgrad (1942,  $M_s \approx 5.1$ ), Velingrad (1977,  $M_s \approx 5.2$ ), the Sofia region (1818 and 1858,  $M_s \approx 5-7$ ) (Dimitrov and Nakov, 2022), and Pernik (1977,  $M_s \approx 5.6$ ) (Dimitrov and Nakov, 2022; Solakov et al., 2020) also show strong spatial correspondence with the identified zones. Furthermore, the majority of earthquakes recorded in Bulgaria between 1981 and 2019 (Solakov et al., 2020) are concentrated within these areas. This demonstrates a strong correlation between the boundaries separating zones of positive and negative vertical tectonic motions and seismic hazard. Consequently, periodic remeasurement of the national levelling network is necessary to monitor tectonic activity and to mitigate the risk of damage to buildings and infrastructure, as well as loss of life, as observed during the largest earthquakes in Bulgaria (Bayliss and Burton, 2007; Matova, 2000). In addition, Figure 9 shows a strong correspondence between the zone boundaries and tectonic depressions, the major river flows in Bulgaria, and the epicentres of the country's largest earthquakes. The estimated distances between them are generally less than 10 km. In more cases, even below 3 km – Plovdiv and Chirpan /Maritsa river/, Kresna and Pernik /Struma river/, Gorna Oryahovitsa, Veliko Tarnovo and Stazhitsa /Yantra river/, Sofia /Iskar river/. These differences can be attributed to the generation of tectonic isolines, residual calculation uncertainties, and seismic and levelling measurement errors.

## 5. Conclusion

This paper introduces a new approach for estimating vertical displacements of the Earth's crust and their rates. The key steps in the procedure include:

- Selecting levelling network values that minimise loop closure errors through separate adjustments for each epoch.
- Reducing the influence of network geometry on final results using IDW.
- Analysing the consistency between the two epoch networks.
- Estimating the vertical displacements and displacement rates of benchmarks.

The result is a new, more accurate and reliable map of recent vertical crustal movements in Bulgaria, which closely aligns with known tectonically active faults. According to the findings, the Morava-Rhodope zone, the Western Balkan Unit, and the Dobrudzha Arch are uplifting, while the Srednogorie Unit, the Moesian Platform (excluding the Dobrudzha Arch), and the Balkan Unit (excluding the West Balkan) are subsiding. Vertical displacement rates range from -3.5 mm/year to +4.0 mm/year, indicating notable tectonic activity. The ratios between the determined Earth's crust annual speeds and their standard deviations, except the benchmarks in Petrich, are in the range between 3.25 and 15.69. In addition, a strong correlation was identified between the boundaries separating zones of positive and negative vertical tectonic motions and the epicentres of the largest earthquakes in Bulgaria. Consequently, these dynamics must be considered when adjusting or establishing national geodetic reference systems, not only to ensure an accurate height reference system but, more importantly, to help mitigate potential loss of life and damage associated with earthquakes.

Because of the fast evaluation of Global Navigation Satellite Systems (GNSS) accuracy (Balodis et al., 2019; Borowski et al., 2025; Celms et al., 2024; Haritonova, 2019), our further research would be dedicated to the concordance between the Earth's vertical motions in Bulgaria obtained by precise levelling and GNSS technologies.

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## Appendix A

### Pseudocode of 3<sup>rd</sup> Selection Procedure

```

INTEGER n, k
INTEGER ARRAY loops
STRING MATRIX Benchmarks
STRING MATRIX Weights
DOUBLE MATRIX Elevations
DOUBLE min_Mu ← +∞
DOUBLE ARRAY best_Elevations
DOUBLE ARRAY Adj_Elevations
STRING Benchmark_RMSE

FUNCTION Constructor(benchmarks, weights, elevations)
  Benchmarks ← benchmarks
  Weights ← weights
  Elevations ← elevations
  n ← number of rows in elevations
  k ← number of columns in elevations
  loops ← new INTEGER[n]
  best_Elevations ← new DOUBLE[n]
  Adj_Elevations ← new DOUBLE[n]
  CALL Calculate(Benchmarks, Weights, Elevations)
END FUNCTION

FUNCTION Adjusted_Elevations()
  STRING result ← empty
  FOR i from 0 to n-1
    append Adj_Elevations[i] formatted to 5 decimals to
result
  END FOR
  RETURN result
END FUNCTION

FUNCTION Calculate(Benchmarks, Weights, Elevations)
  CALL GenerateCombinations(0)
END FUNCTION

/*
This generates all possible combinations (k^n).
WeightedLevelling(Weights, Benchmarks) performs the
geometric levelling adjustment.
*/

```

```
FUNCTION GenerateCombinations(current)
  IF current = n THEN
    adj ← new WeightedLevelling(Weights, Benchmarks)
    FOR i from 0 to n-1
      adj.Elevations[i,0] ← Elevations[i, loops[i]-1]
    END FOR
    CALL adj.Calculate()

    IF adj.Mu < min_Mu THEN
      min_Mu ← adj.Mu
      FOR i from 0 to n-1
        best_Elevations[i] ← adj.Elevations[i,0]
        Adj_Elevations[i] ← adj.Adj_Elevations[i,0]
      END FOR

      Benchmark_RMSE ← adj.BenchMark_RMSE()
      BUILD output summary string
      (timestamp, Mu, best_Elevations,
      adj.Elevations, Benchmark_RMSE)
    END IF

    RETURN
  END IF

  FOR counter from 1 to k
    loops[current] ← counter
    CALL GenerateCombinations(current + 1)
  END FOR
END FUNCTION
```

## Appendix B

### The Second Levelling of Bulgaria data that were used in the study

From	To	Length (km)	Height differences in (m)		
			I	II	Mean
Knezha	Vidin	163.168	-119.33810	<b>-119.32692</b>	-119.33251
Zlatitsa	Knezha	117.062	-513.28172	<b>-513.31777</b>	-513.29975
Sofia	Zlatitsa	93.805	158.75188	<b>158.78483</b>	158.76836
Vidin	Sofia	255.104	473.83649	473.88345	<b>473.85997</b>
Knezha	Levski	104.418	<b>-90.92189</b>	-90.91337	-90.91763
Levski	Kazanlak	174.666	305.97630	<b>306.00681</b>	305.99156
Zlatitsa	Kazanlak	127.928	-298.25715	<b>-298.23291</b>	-298.24503
Levski	Tutrakan	172.996	<b>-35.26736</b>	-35.21556	-35.24146
Tutrakan	Varna	211.602	<b>30.17987</b>	30.18738	30.18362
Varna	Burgas	136.621	<b>17.09283</b>	17.12000	17.10642
Burgas	G. Alexandrovo	71.044	<b>71.66161</b>	71.65542	71.65852
G. Alexandrovo	Kazanlak	133.286	<b>222.34934</b>	222.38840	222.36887
Elhovo	Burgas	114.001	<b>-46.01353</b>	-46.02272	-46.01813
Elhovo	G. Alexandrovo	69.470	25.64586	<b>25.65108</b>	25.64847
Momchilgrad	Elhovo	253.155	-122.37364	-122.30770	<b>-122.34067</b>
Momchilgrad	Kazanlak	171.824	125.63275	<b>125.66310</b>	125.64792
Momchilgrad	Dospat	201.120	<b>945.51263</b>	945.51825	945.51544
Dospat	Zlatitsa	177.768	<b>-521.60987</b>	-521.60246	-521.60617
Dospat	Petrich	146.882	-1085.97929	<b>-1085.92241</b>	-1085.95085
Petrich	Sofia	260.335	405.47898	<b>405.52389</b>	405.50144

Note: The bolded values are the height difference values selected by  $3^{20}$  adjustments. They produce adjustment with the least aposteory standard error.

## Appendix C

### The Third Levelling of Bulgaria data that were used in the study

From	To	Length (km)	Height differences in (m)		
			I	II	Mean
Knezha	Vidin	185.971	-119.30612	<b>-119.32912</b>	-119.31762
Zlatitsa	Knezha	126.510	<b>-513.28901</b>	-513.32191	-513.30546
Sofia	Zlatitsa	95.510	158.78266	<b>158.76916</b>	158.77591
Vidin	Sofia	406.478	<b>473.84495</b>	473.81374	473.82935
Knezha	Levski	112.309	<b>-90.93347</b>	-90.93774	-90.93561
Levski	Kazanlak	184.355	<b>306.00943</b>	305.95973	305.98458
Zlatitsa	Kazanlak	126.260	<b>-298.21085</b>	-298.20101	-298.20593
Levski	Tutrakan	191.096	<b>-35.20747</b>	-35.21508	-35.21128
Tutrakan	Varna	225.754	<b>30.14512</b>	30.11246	30.12879
Varna	Burgas	151.930	17.06735	<b>17.07590</b>	17.07162
Burgas	G. Alexandrovo	71.505	71.65568	71.62065	<b>71.63817</b>
G. Alexandrovo	Kazanlak	139.409	222.36740	222.32509	<b>222.34625</b>
Elhovo	Burgas	307.110	-45.92758	<b>-45.96507</b>	-45.94632
Elhovo	G. Alexandrovo	66.459	<b>25.65452</b>	25.65348	25.65400
Momchilgrad	Elhovo	261.637	<b>-122.39892</b>	-122.38961	-122.39427
Momchilgrad	Kazanlak	184.636	<b>125.57867</b>	125.56773	125.57320
Momchilgrad	Dospat	203.882	945.55511	<b>945.51450</b>	945.53481
Dospat	Zlatitsa	190.734	-521.67446	<b>-521.71862</b>	-521.69654
Dospat	Petrich	147.738	-1085.96916	<b>-1085.97044</b>	-1085.96980
Petrich	Sofia	204.984	<b>405.48552</b>	405.45665	405.47108

Note: The bolded values are the height difference values selected by  $3^{20}$  adjustments. They produce adjustment with the least a posteriori standard error.

## Appendix D

### Adjusted benchmark heights in the Second Levelling of Bulgaria after 3<sup>n</sup> selections and IDW(p), where $p = \{4, 5, 6, 7, 8\}$

Benchmark	p = 4	p = 5	p = 6	p = 7	p = 8
Burgas	82.46714	82.46740	82.46760	82.46775	82.46785
G. Alexandrovo	154.12905	154.12923	154.12936	154.12946	154.12953
Elhovo	128.47827	128.47836	128.47842	128.47846	128.47850
Tutrakan	35.20078	35.20142	35.20201	35.20253	35.20297
Kazanlak	376.47792	376.47817	376.47839	376.47857	376.47871
Levski	70.47055	70.47098	70.47135	70.47164	70.47185
Momchilgrad	250.81400	250.81412	250.81428	250.81446	250.81463
Dospat	1196.32312	1196.32311	1196.32312	1196.32313	1196.32311
Zlatitsa	674.71084	674.71105	674.71126	674.71145	674.71160
Knezha	161.39269	161.39302	161.39332	161.39357	161.39376
Vidin	42.06580	42.06612	42.06641	42.06665	42.06684
Sofia (N. Iskar)	515.92599	515.92621	515.92643	515.92662	515.92677
Petrich	110.40084	110.40079	110.40077	110.40075	110.40073

### Adjusted benchmark heights in the Third Levelling of Bulgaria after 3<sup>n</sup> selections and IDW(p), where $p = \{4, 5, 6, 7, 8\}$

Benchmark	p = 4	p = 5	p = 6	p = 7	p = 8
Burgas	82.45258	82.45222	82.45194	82.45173	82.45157
G. Alexandrovo	154.09076	154.09040	154.09012	154.08990	154.08973
Elhovo	128.43627	128.43589	128.43560	128.43538	128.43521
Tutrakan	35.22314	35.22234	35.22153	35.22077	35.22010
Kazanlak	376.43660	376.43654	376.43639	376.43622	376.43606
Levski	70.42703	70.42643	70.42584	70.42533	70.42493
Momchilgrad	250.85287	250.85357	250.85406	250.85442	250.85469
Dospat	1196.36637	1196.36640	1196.36630	1196.36616	1196.36602
Zlatitsa	674.64831	674.64807	674.64775	674.64743	674.64715
Knezha	161.36005	161.35960	161.35912	161.35869	161.35833
Vidin	42.03107	42.03055	42.03004	42.02958	42.02921
Sofia (N. Iskar)	515.87922	515.87895	515.87861	515.87828	515.87800
Petrich	110.39546	110.39555	110.39552	110.39545	110.39537

**Estimated vertical speeds in mm/year of the nodat benchmarks under different values of the power parameter  $p$ , applied in the IDW step, where  $p = \{4, 5, 6, 7, 8\}$**

<b>Benchmark</b>	<b><math>p = 4</math></b>	<b><math>p = 5</math></b>	<b><math>p = 6</math></b>	<b><math>p = 7</math></b>	<b><math>p = 8</math></b>
Burgas	-0.63	-0.65	-0.67	-0.69	-0.70
G. Alexandrovo	-1.65	-1.67	-1.69	-1.70	-1.71
Elhovo	-1.81	-1.83	-1.84	-1.86	-1.86
Tutrakan	0.96	0.90	0.84	0.79	0.74
Kazanlak	-1.78	-1.79	-1.81	-1.82	-1.84
Levski	-1.87	-1.92	-1.96	-1.99	-2.02
Momchilgrad	1.67	1.70	1.71	1.72	1.73
Dospat	1.86	1.86	1.86	1.85	1.85
Zlatitsa	-2.69	-2.71	-2.74	-2.76	-2.78
Knezha	-1.41	-1.44	-1.47	-1.50	-1.53
Vidin	-1.50	-1.53	-1.57	-1.60	-1.62
Sofia (N. Iskar)	-2.01	-2.04	-2.06	-2.08	-2.10
Petrich	-0.23	-0.23	-0.23	-0.23	-0.23

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