

# Method for Sensor Data Alias-free Acquisition from Wideband Signal Sources and their Asymmetric Compression-Reconstruction

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**Abstract.** A method for acquisition of sensor data from wideband signal sources with their asymmetric compression-reconstruction is proposed and discussed. It is based on using a special type of pseudo-randomized non-uniform sampling. Compression of data representing the involved wideband signals is achieved by taking signal sample values at a rate significantly lower than the Nyquist rate. While the acquired data are not specially processed for their compression, the signal sample value taking has to follow specific requirements defined in a way ensuring: (1) the possibility of alias-free reconstruction of these signals; (2) elimination of the cross-interference between the signal components. Reconstruction of the sensor data compressed in the suggested way represents the basic computational burden. Computer simulation results of the considered method are given.

**Keywords:** digital signal processing, sub-Nyquist sampling, sensor networks.

## 1. Introduction

The basic function that has to be fulfilled for compression of acquired sensor data is digitizing the original signal in a way providing for compact digital representation of the original signal. This digital representation, in the format of a sparse sequence of numerical values, is to be constructed either as specifically taken signal sample values or as some other numerical quantities formed at the stage of signal data compression by transforming the digital signal. The compressed data then might be transmitted over communication channels or stored in a memory.

A method for sensor data compression, belonging to the class of methods based on signal transforms performed at the stage of data compression, is described in (Baraniuk (2007)). Some methods involving distributed and compressed sensing also have been proposed (Baron, Wakin, Duarte, Sarvotham, Baraniuk (2005), Kumar, Ishwar, Ramchandran (2004)). A typical feature of the data compression-reconstruction techniques of this class is their symmetry in the sense that the acquired data are processed at both ends of each data compression and reconstruction cycle. Consequently, there

should be sufficient resources for performance of these data conversions at both ends of the data acquisition-compressing-transmitting-reconstructing chain. Under certain conditions, that clearly represents a serious problem.

Attention is drawn to the possibility of avoiding this particular drawback. A specific approach to this data compression problem is suggested and discussed in this paper. It is based on the theory and techniques of randomized signal sampling and processing. A lot of work has been done in that area over a long period of time. The achieved theoretical and practical results of randomized signal processing are described and discussed in (Bilinskis, Mikelsons (1992)). While this approach to signal digitizing and processing offers a lot of various options in the area of processing signals digitally, basically it is focused on enlarging the frequency range for Digital Alias-free Signal Processing (DASP) (Bilinskis (2007), Artyukh, Bilinskis, Sudars, Vedin (2008)). As the DASP technology has the capability of avoiding frequency overlapping or aliasing of signal components in the cases of sub-Nyquist signal sampling, the DASP methods, algorithms and techniques are also applicable in the area of compressive sensing.

The data compression-reconstruction techniques based on deliberately randomized sub-Nyquist sampling of signals are well suited and attractive for sensor data acquisition and compression applications. The apparent advantage of these techniques is the fact that the computational operations that are carried out at the stage of data acquisition from relatively wideband signal sources are much simpler in comparison with those that have to be performed at the stage of the signal reconstruction from the compressed data. Thus the data compression techniques based on DASP are typically asymmetric. Compression of data performed in accordance to these techniques actually does not require performance of computations at all. Sample values from the signals simply have to be taken in a way strictly following certain rules.

Definitions of the rules to be followed at dropping out certain signal sample values at the stage of data compressing depend on the specific sampling methods that are used. While there are some mandatory requirements that have to be met in all cases, there are also some variable components in those rules regulating the data acquisition and compression procedures. Sub-Nyquist sampling, obviously, in all cases has to be carried out in a way providing for elimination or at least sufficient suppression of signal component overlapping or aliasing. That means the signal sample values always have to be taken non-equidistantly. In other words, the sampling process always has to be nonuniform as only under that condition it is possible to avoid frequency overlapping and to ensure alias-free wideband signal reconstruction in the cases where such signals are represented by sparse sequences of their sample values taken at a sub-Nyquist mean sampling rate. However various approaches to implementation of nonuniform sampling are possible. It is another matter which of the specific nonuniform sampling options is better suited for the considered application.

The so-called additive pseudo-randomized sampling (Bilinskis, Mikelsons (1992), Bilinskis (2007), Artyukh, Bilinskis, Sudars, Vedin (2008)) is the first option that comes to mind when a promising method for nonuniform sampling is looked for under the given general conditions for compressive sensing. It is briefly discussed in Section II. However, implementation of this approach requires performance of relatively complicated computations that have to be carried out at the stage of the compressed signal reconstruction (Artjuhs, Bilinskis, Rybakov (2008)). Apparently it is essential to achieve a relative simplicity also for the procedures involved in the recovery of the original signal from the compressed data. An approach making it possible is suggested and described in Section III. This approach and the method based on it actually represent the basic topic of

this paper. Data acquisition from wideband signal sources is considered not only because wideband signals have to be digitized and processed in many various applications. Alias-free digital processing of wideband signals is also more demanding than processing of other types of signals. This fact has been taken into account at research and development activities related to this paper. The developed and proposed method for pseudo-randomized nonuniform sampling, well suited for asymmetric sensor data compression-reconstruction, has been MATLAB simulated and the obtained results illustrate the basic principles of it. They are discussed in the following Sections IV and V. Studies of the proposed method in detail are out of scope of this paper. The discussion on applications of the method for wireless sensor networks is presented in Section VI. While this method is recommended for acquisition of sensor data from wideband signal sources with their asymmetric compression-reconstruction, there are also other methods for alias-free signal sampling that can be used for this type of compressed data acquisition. Specifically, we refer, for example, to the method for adapting signal processing to the sampling non-uniformities described in Bilinskis, I. (2007).

## 2. Using of Pseudo-randomized Additive Sampling

To implement additive random sampling (Bilinskis, Mikelsons (1992), Bilinskis (2007)), signal sample values are taken at time instants:

$$t_k = t_{k-1} + \tau_k \quad k=0, 1, 2, \dots \quad (1)$$

where  $\tau_k$  is a random variable.

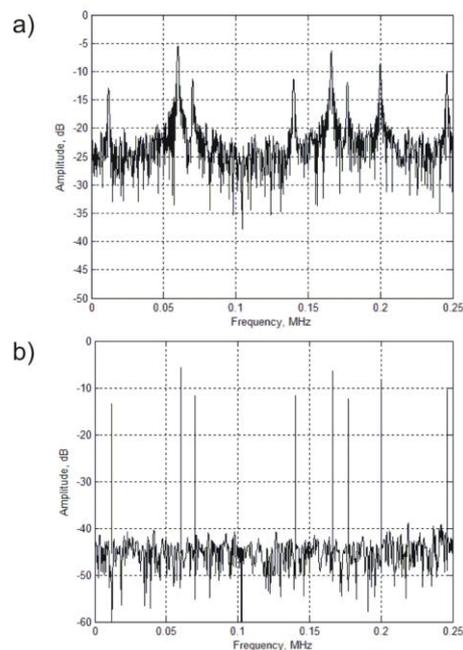
The nonuniform intervals  $\{\tau_k, \tau_{k+1}\}$  between successive sampling events are characterized first of all by their mean value  $\mu$  and the standard deviation  $\sigma$ . The mean sampling rate is equal to  $1/\mu$  and the value of  $\sigma$  reflects the randomness introduced into the sampling process.

This randomized sampling scheme is called additive because the randomness present in the sampling process accumulates in time. And that property actually is very useful. If the random variables used are statistically independent and identically distributed, then sampling of all signal instantaneous values, due to this property, takes place with constant probability even if the distribution of  $\tau_k$  varies in large margins. Therefore, the power of the randomness introduced into the sampling process might be to some extent controlled by varying the parameter  $\sigma$ . That often proves to be useful as the positive anti-alias capability usually might be achieved at small  $\sigma$  values and minimizing of them helps to reduce some negative effects like statistical errors in processing the digital signals obtained in result of this kind of sampling.

The additive sampling process might be both either randomized or pseudo-randomized. The latter version is more appropriate for sensor data compression applications. As it is shown in (Bilinskis, Mikelsons (1992), Bilinskis (2007)), the signal sample values then are locked to a regular time grid. Therefore all parameters of such sampling process, like the mean value  $\mu$  and the standard deviation  $\sigma$ , then are given as digital quantities with the smallest time digit equal to the period  $\delta$  of the respective periodic clocking pulse sequence. This pseudo-randomized sampling actually might be interpreted in terms of a periodic sampling process with subsequent pseudo-random taking out some quantity of the periodically obtained sample values. The frequency of this periodic sampling is equal to  $1/\delta$ . Therefore the highest frequency that might be present in

the signal spectrum sampled in this way should be at least two times lower than  $1/\delta$  to avoid full scale aliasing. As a part of the signal sample values are pseudo-randomly taken out, the mean sampling rate  $1/\mu$  is significantly lower. Some frequency overlapping processes (considered in (Bilinskis (2007)) as fuzzy aliasing), tied to the mean sampling rate  $1/\mu$ , are observed. The ratio of the parameters  $1/\delta$  and  $1/\mu$  characterize the initial periodic and the reduced signal sampling rates, respectively. How much the sampling rate might be reduced and what would be the properties of the signal sample value sequence left over after the pseudo-random decimating of the initial periodic sample value flow depends both on the parameters of the respective analog signal digitized in this way and on the procedure used at the mentioned process of dropping out selected signal sample values. This procedure, in general, has to be executed so that the remaining sample value sequence would have the properties typical for additive sampling. The sampling rate reduction, achievable on the bases of the considered pseudo-randomized sampling, is significant. For example, reduction of the sampling rate up to 30 times have been achieved in the described way at digital signal analysis in the frequency range up to 1.2 GHz while the mean sampling rate were only about 80 MS/sec (Bilinskis (2007)). The achievable data compression rates depend on the properties of the analog signal, including the dynamics of it, characterized by the parameters of its non-stationarity.

What happens when data compression has been performed according to the additive sampling approach is illustrated in Figure 1.

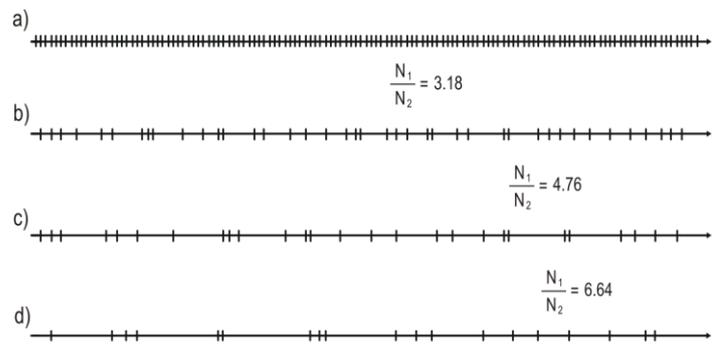


**Fig. 1.** Amplitude spectrum of a signal obtained in the case of additive pseudo-randomized sampling, (a) before and (b) after adapting the performed DFT to the non-uniformities of the involved sampling process

The amplitude spectrum of the signal, given in Figure 1a and obtained by DFT of the pseudo-randomly sampled signal, is buried in the background noise. However, in fact, it is not a random noise and it is not even a random process either. Actually this process, looking like a noise, is result of a specific aliasing process called fuzzy aliasing (Bilinskis (2007)). Peak frequencies of it might be tracked down and tied to the frequencies of specific signal components. This fuzzy aliasing appears in result of the cross-interference taking place whenever analog signals are sampled nonuniformly. Consequently, to take out this fuzzy aliasing overlapping the true signal spectrum, something has to be done to eliminate first this cross-interference between the signal components. It has been discovered how to do that (Bilinskis (2007), Artjuhs, Bilinskis, Rybakov (2008)). However the involved computations are resource and time consuming. To overcome this drawback, a specific approach to nonuniform sampling has been developed and is proposed for the compressive sensing applications. Note that although the proposed sampling method also is nonuniform and it might be described in terms of pseudo-randomized sampling, actually there is nothing random there. This sampling model is fully deterministic.

### 3. Multi-periodic Sampling Model

As it shown in (Bilinskis (2007)), the pseudo-randomized additive sampling point processes (sequences of signal sample taking time instances) might be decomposed into a number of periodic point processes with pseudo-randomly skipped points. It is a significant fact that helps to understand the essence of the mentioned cross-interference. To avoid this harmful effect, a nonuniform sampling point process not containing components with randomly skipped points has to be designed. That has been done. Application of such a point process for nonuniform sampling confirmed that indeed it is possible to perform nonuniform sampling without harmful cross-interference between components of the sampled in this way signals. Three nonuniform sampling point processes of this type are shown in Figure 2 (b), (c) and (d) in comparison with the basic periodic point process given in Figure 2 (a).



**Fig. 2.** Examples of specific nonuniform sampling point processes (b), (c), (d) and the basic periodic sampling point process (a)

The difference between the given three point processes is in their differing mean sampling rates. Their sampling rates are reduced in comparison with the rate of the given basic process 3.18, 4.76 and 6.64 times, respectively. All of the illustrated sampling point processes are composed from 3 periodic point processes with frequencies selected so that periods of all these particular point processes contain an integer number of a constant small time interval  $\delta$ . Actually this time interval  $\delta$ , as the smallest period of the respective periodic clocking pulse frequency  $1/\delta$ , limits the highest frequency of the input signal, it has to be at least two times smaller. If a signal is observed during a time interval  $\Theta$ , then the respective digital signal contains  $N^{(1)}, N^{(2)}, N^{(3)}$  sample values taken periodically at frequencies  $f_1, f_2$  and  $f_3$ . That is taken into account at DFT of these signal sample values.

As the sampling point process is composed from three periodic sampling point processes, aliasing of the obtained digital signal components occur according to three aliasing frequency rows:

$$f_v, f \pm f_v, 2f \pm f_v, 3f \pm f_v, \dots \quad (2)$$

That helps to identify and eliminate the various aliases.

#### 4. Recovery of the Original Signal Spectrum

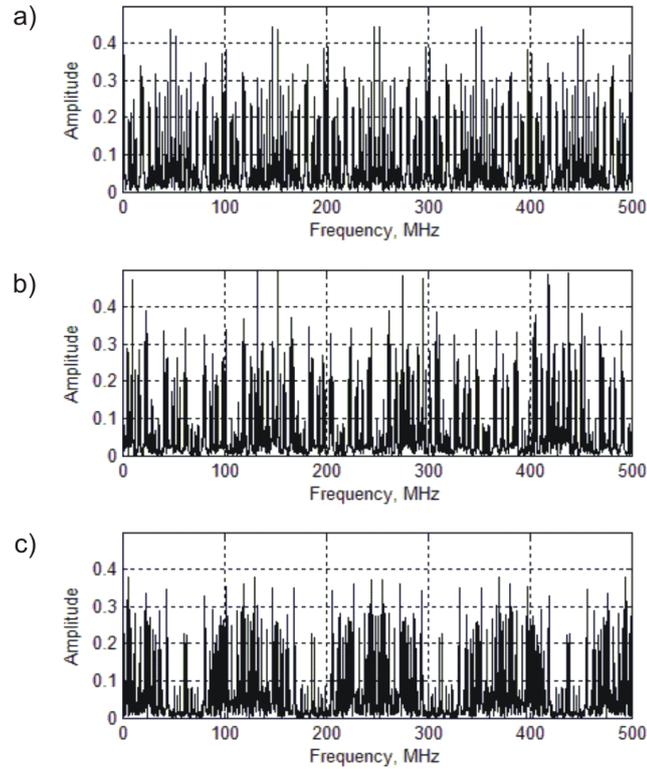
Recovery of the original signal spectrum and reconstruction of its waveform in the time domain from the nonuniform signal sample value sequence are based on Discrete Fourier Transforms of the nonuniformly sampled signals. These transforms are carried out separately for all three periodic components of the sampling point stream as follows:

$$\begin{aligned} a_i^{(j)} &= \frac{2}{N^{(j)}} \sum_{k=1}^{N^{(j)}} x(t_k^{(j)}) \cos \omega_i t_k^{(j)} \\ b_i^{(j)} &= \frac{2}{N^{(j)}} \sum_{k=1}^{N^{(j)}} x(t_k^{(j)}) \sin \omega_i t_k^{(j)} \end{aligned} \quad (3)$$

Where  $j = 1, 2, 3$ ;  $k = 1, 2, 3 \dots N$ .

The estimates of Fourier coefficients, obtained in result of direct DFT of a band limited nonuniformly sampled signal, reflect the features of both the signal and the involved sampling point process. The problem is that in this case sampling of the signal is performed at time instants dictated by composition of three periodic processes and overlapping of signal components take place according to (2).

$$A_i^{(j)} = \sqrt{(a_i^{(j)})^2 + (b_i^{(j)})^2} \quad (4)$$



**Fig. 3.** Spectrograms (a), (b) and (c) calculated on the basis of the estimated first, second and third sets of the Fourier coefficients obtained from the respective three periodic sub-sets of the sampling point stream

Three spectrograms are calculated from the obtained three sets of the Fourier coefficients. Evidently the estimated Fourier coefficients provide the information for calculation of the basic parameters (amplitudes and phase angles at the considered frequencies) of the respective signal components. Due to the different conditions for aliasing, the obtained spectrograms differ substantially.

A typical example of the spectrograms obtained at this signal reconstruction stage is given in Figure 3.

As can be seen, the spectrograms calculated at this stage differ significantly. To recover the true signal amplitude spectrum, the true signal components have to be identified and separated from the aliases. That is done by exploiting the fact that one and the same set of the true signal components is present in all three of these particular spectrograms while only the aliases are different. Specifically, the averaged spectrogram is calculated as:

$$A_{i \text{ mean}} = \frac{1}{J} \sum_{j=1}^J A_i^{(j)} \quad (5)$$

Where  $J=3$  in the considered case but more than 3 periodic components might be used for composing the nonuniform sampling point stream.

Then all particular spectrograms are summed up and the maximum peak values of the resulting spectrogram are fixed as

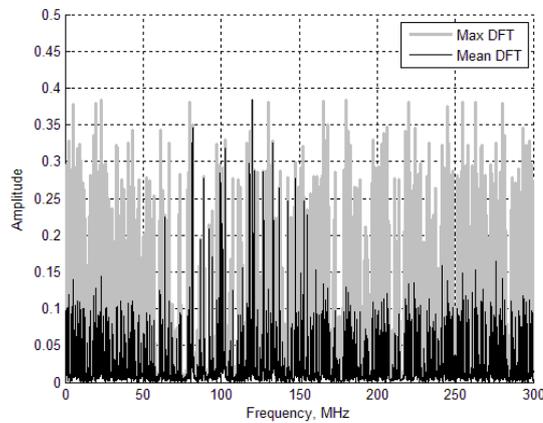
$$A_{i \max} = \max(A_i^{(1)}, A_i^{(2)}, A_i^{(3)} \dots A_i^{(J)}) \quad (6)$$

The differences:

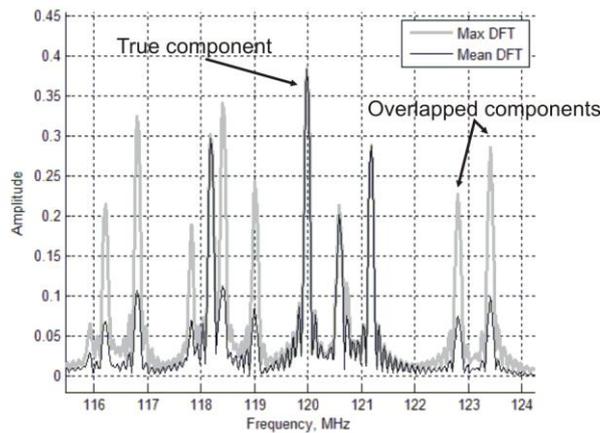
$$\Delta A_i = A_{i \max} - A_{i \text{ mean}} \quad (7)$$

These differences are used for discovery of the true signal components. They are close to the points on the frequency axis where  $\Delta A_i \rightarrow 0$ .

Figures 4 and 5 illustrate this approach to the recovery of the true signal spectrum.

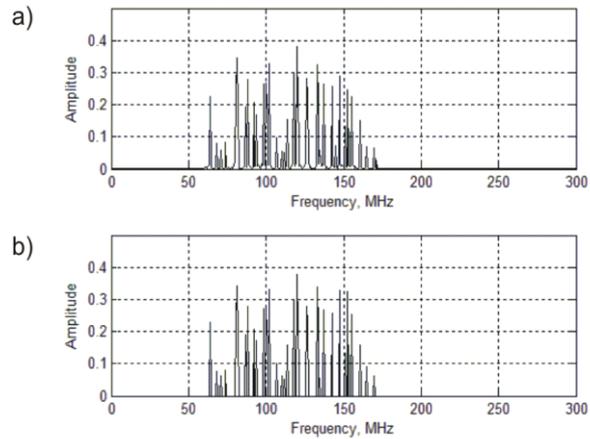


**Fig. 4.** Example of the averaged and summary spectrograms



**Fig. 5.** Zoomed up segment of the spectrogram given in Figure 4

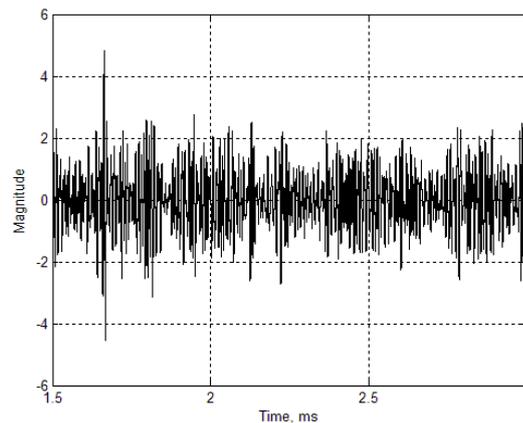
The signal spectrogram recovered in the case of the discussed example is given in Figure 6 together with the true signal spectrogram. They are close.



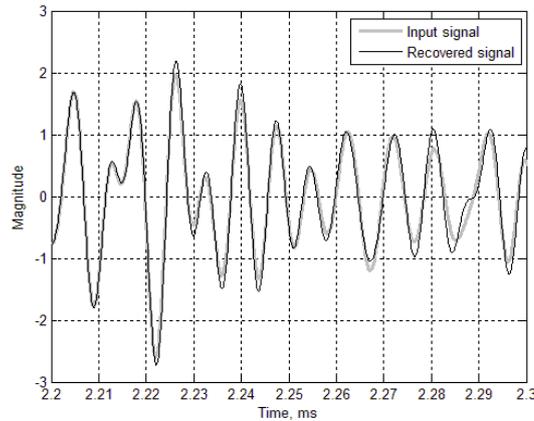
**Fig. 6.** True (a) and the recovered (b) signal spectrograms

## 5. Recovery of the Original Signal

The recovered signal spectrogram can be used for reconstruction of the original signal waveform on the basis of the inverse Fourier transform. The results of such waveform reconstruction, obtained under conditions of the considered example for the signal waveform of Figure 7, are given in Figure 8.



**Fig. 7.** Waveform of the considered signal



**Fig. 8.** Reconstructed waveform in comparison with the true waveform of the signal

As can be seen in Fig.8, the achieved accuracy of the waveform reconstruction in this case seems to be acceptable.

The achievable reduction of the mean sampling rate at sensor data acquisition depends, in general, on the structure of the signal, required accuracy of the signal reconstruction and the dynamics of the respective signals. It is not directly limited by the upper frequency in the signal spectra. Studies of the proposed method in more detail are beyond the scope of this paper.

## 6. Discussion – Application to Sensor Networks

The proposed method is intended to considerably lower the sampling rate of an analog signal, thus allowing the digital processing of high frequency signals. Usually sampling of signals at MHz or even GHz rates is inconceivable in sensor networks that are battery or environmentally powered due to high energy consumption. Also, the sensor node complexity and cost is elevated in such cases due to higher frequencies requiring specific ADC and processing devices. For example, a special purpose hardware node was developed and adapted for wildlife tracking using several microphones and audio signals (Liu, Matic, Berkeley, (2006), Ali, Yao, Collier, Taylor, Blumstein, Girod (2007)), or special purpose software for transcoding high data rate signals (Greenstein, Pesterev, Mar, Kohler, Judy, Farshchi, Estrin (2005)).

With our method it is possible to reduce sampling rate 20-30 times, as shown in (Bilinskis (2007)). Even if the raw sampled data is just being forwarded over a wireless channel, this reduces the data rate by the same amount, leading to significant energy savings by the transceiver component of the node thus increasing the node life time. On the other hand, it is possible to process high frequency signals onboard a sensor node because of the method and fewer sampling points and transmit just the processing result. In the future, we plan to investigate the applications of the method described in this paper to physical world sensor networks challenging the both aspects – the high frequency signal processing and the low energy consumption. We anticipate implementations in software, firmware, and are considering prototyping specific data acquisition devices featuring pseudo random sampling.

## 7. Conclusions

1. A method is proposed for sensor data alias-free nonuniform acquisition from wideband signal sources at a reduced sub-Nyquist mean sampling rate.
2. This method is based on a specific multi-periodic sampling point process model and other techniques typical for Digital Alias-free Signal Processing (DASP).
3. Sensor data compression according to the described approach to data acquisition does not require performance of computations at the stage of the data flow compression. The computational burden in the compression-reconstruction procedures is put fully on the original signal reconstruction. Therefore this data compression-reconstruction method is typically asymmetric in that sense.

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