# On Multi-Objective Optimization Heuristics for Nurse Rostering Problem

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**Abstract.** The nurse rostering problem is usually tackled in the relevant literature as a singleobjective optimization problem. Recently, however, the attention of researchers has been attracted by a multi-objective version of that problem. In the present paper we propose an algorithm for the multi-objective version of the problem. The performance of the proposed algorithm is compared with that of the simulated annealing method. The results of the application of the considered algorithms to a real-world optimization problem are provided.

Keywords. Nurse rostering problem, shift sequence, multi-objective optimization.

# 1. Introduction

Manual scheduling is a quite frustrating task for planners and persons making schedules for the health care personnel, because many objectives, like workload distribution, personnel coverage, personal preferences, etc. should be taken into consideration. Since the personnel scheduling involves human resources it is important to maintain the high satisfaction of personnel in their working environment, i.e. "friendly schedule", paying as much as possible attention to their scheduling preferences. Some studies prove that such factors as the shift length, days of week worked, and hours worked give the direct impact on the personnel stress (Purcell et al., 2011). Other studies prove the direct relationship between the personnel burnout and low satisfaction got from patients they take care of (Vahey et al., 2004). Therefore it is not surprising that many approaches have been proposed to deal with the automated scheduling problems. However, it should be emphasized that these problems are hard to solve since they belong to the class of NP-hard problems (Shul'gina, 2004; Brucker et al., 2011; Hadwan et al., 2013; Arora and Barak, 2009). Let us start with a review of some most popular methods. The real-life health care personnel scheduling problems (usually they are called either the nurse scheduling or nurse rostering) are difficult to solve since they are highly constrained by various laws, rules, and preferences.

The problem considered in this paper is based on real world data from one of the major hospitals in Lithuania. The scheduling period (planning horizon) is one month. The problem is to assign working shifts and days off for each nurse so that the assigned workload is as closely aligned as possible to the workload defined in the nurses'

contracts (henceforth in this text, workload defined by a contract will be called required workload). Also henceforth in this text, the term workload balance will be used quite a lot. By using this term we mean the difference between required workload and real assigned workload. Ideal balance is said to be achieved when the value of workload balance is equal to zero.

As described in (Aickelin and Dowsland, 2000) the problem can be decomposed into three independent stages. The first stage uses a knapsack model to ensure that sufficient nurses are available to meet the covering constraints; if that constraint cannot be satisfied, additional nurses are allocated to the ward, so that the problem tackled in the second phase is always feasible. The second stage involves allocating for the actual days and nights to be worked by each nurse. The third stage allocates shifts on days, allocated in the second stage.

The second stage problem usually is formulated as an integer linear program. The problems of such a type seem favourable for application of genetic algorithms (GAs). Experimental investigations, however, have shown that for some instances even finding feasible solutions by GAs is difficult, and an improvement of the performance was attempted by means of the hybridization of GAs with hill climbers (Aickelin and Dowsland, 2000). Further improvements have been made using GA coupled with a decoding routine (Aickelin and Dowsland, 2004) that transforms the genotype of a string into phenotype. The genotypes are encoded as permutations of nurses for its simpler nature rather than the permutations of shifts. The decoding routine builds schedules for the nurses from encoded list. The decoding routine consists of three sub-routines (decoders). The first one considers feasibility of solutions, the second one considers the nurses' preferences, and the third one combines the first two decoders. The results of experiments pointed out that the decoding routine alone is not capable to find feasible solutions (only less than 5% solutions found are feasible). The decoding routine combined with GA improves results with respect to the feasibility and overall cost (Aickelin and Dowsland, 2004). An algorithm proposed in (Aickelin and Li, 2007) is based on Bayesian networks, and it outperforms the above mentioned method proposed in (Aickelin and Dowsland, 2004).

The nurse scheduling problem (Burke and Soubeiga, 2003) was also tackled by the means of a hyperheuristic where tabu type search is used to select an available heuristic similarly as in a quiz game: at every step the score of the contestant is increased in case a right answer is given, and is decreased in case of the wrong answer. The heuristic with the highest score is selected, in case there are several heuristics having the highest score the heuristic is selected randomly. The investigation revealed that proposed mechanism is more effective when using long tabu list combined with low rates of negative reinforcement.

Combination of genetic algorithm GA and simulated annealing (SA) (Torn and Zilinskas, 1989) method have been proposed for solving nurse rostering problem (Bai et al., 2010). Genetic algorithm is known for its capability of searching large search spaces but is less effective in identifying local optima and in order to improve effectiveness of GA, the SA heuristic have been emerged into GA. The combination of those two algorithms outperforms the separately used algorithms.

Completely different approach has been proposed in (Brucker et al., 2010) where a schedule for each nurse is constructed using the shift sequences, rosters are reconstructed iteratively, and the best roster found so far is memorized.

Recently a harmony search algorithm (HSA) has been adapted to solve the nurse scheduling problem (Hadwan et al., 2013) where the authors claim that HSA

outperforms GA, the shift sequence based algorithm (Brucker et al., 2010), and in some cases also the scatter search (Burke et al., 2010). Recent studies (Liogys, 2012) show that SA method outperforms the shift sequence based method with respect to the calculation time.

Although many experimental results have been published, the selection of the most suitable algorithm for the applied problem is still problematic since the performance of various heuristic algorithms crucially depends on the problem's data. The nurse scheduling problem with data corresponding to one of the largest Lithuanian hospital is considered in (Liogys and Zilinskas, 2012) where the hybrid method combining SA with a shift sequence based method has been shown most suitable for such type of problems. Nevertheless some difficulties cannot be surmounted because of the concept of single objective optimization where some objectives are transformed into constraints. The so called soft constraints in most cases are formulated in this way involving much of subjectively. The application of the multi-objective concept in the formulation of the problem seems here promising.

Several methods of multi-objective nurse scheduling problem have been published recently. The attention of the authors of the review in (Suman and Kumar, 2006) is focused on the multi-objective version of the SA algorithms adapted to the nurse scheduling problems. The authors came up with conclusions that unambiguously to tell which one of the algorithms is the best is impossible, because the efficiency of the algorithm depends on the specific problem.

In the paper (Burke et al., 2012) the authors compare traditional way of solving multi-objective optimization problem, combining multiple objectives to single objective using weighted-sum function, with their own proposed domination-based evaluation function. Test results point out that the proposed approach produces much more Pareto optimal solutions than the traditional weighted-sum function.

The branch and price algorithm has been adapted to solve multi-objective nurse scheduling problem (Maenhout and Vanhoucke, 2010). The authors investigated that combination with 0-1 branching strategy selecting the worst fractional assignment with the branching strategy performs well. Also pruning strategies are reviewed for reduction of branch and bound tree.

The recent research (Liogys and Zilinskas, 2012) shows that enhanced shift sequence based method outperforms the shift sequence based method for solving a single objective scheduling problem. The present paper investigates the efficiency of these two methods adapted to solving a multi-objective nurse scheduling problem, compares it with the efficiency of the simulated annealing method (SMOSA) (Suman and Kumar, 2006), and identifies which of the methods is the most suitable to solve multi-objective nurse scheduling problem when workload balance as the key objective is considered.

Typically, a nurse scheduling problem described in scientific literature is in some ways (scheduling period, set of constraints, shift types, etc.) distinct among other nurse scheduling problems. Almost all articles about nurse scheduling problem tackle this problem using different approach. Despite big variety of modern approaches proposed genetic algorithms and simulated annealing based approaches remain the most favourite among scientists and researchers. However, the research (Kundu et al., 2008) points out the fact that simulated annealing outperforms the genetic algorithm with respect to computational time and the quality of the schedules. With respect to these research results we have selected comparison of our solutions with simulated annealing approach, in this article.

# 2. Multi-Objective Optimization Based Nurse Rostering

As mentioned above, the problem considered here is based on real world problem, i.e. we use data of one of the major hospitals in Lithuania. We aim at assigning working shifts and days off for each nurse in scheduling period (one month) that the workload would satisfy the government and hospital regulations.

Additional requirements to the problem are: to distribute certain types of shifts (night and watch) as fairly as possible among nurses; also, to satisfy various personal preferences as much as possible.

In our problem there are four types of shifts (listed in Table 1) available to assign to nurses:

rabler. Shift types.		
Abbreviation	Description	Period
М	Morning shift	07:30 - 15:12
D	Day shift	15:12 - 18:48
Ν	Night shift	18:48 - 09:12
W	Watch shift	08:00-08:00

Table1. Shift types.

Night shift starts at 18:48 and ends at 09:12 next day. Watch shift starts at 08:00 and ends at 08:00 next day.

Nurse scheduling problems are highly constrained combinatorial problems (Hadwan et al., 2013). Constraints considered in such problems are usually of two types: hard constraints, i.e. those which must be satisfied; and soft constraints, i.e. those which preferably should be satisfied, but are not mandatory. In a multi-objective optimization case, each constraint or group of soft constraints is usually treated as objectives. Usually the planner planning the schedule considers some objectives as very important and some objectives as not that important; which ones depend on the particular situation. In (Burke et al., 2012) the authors considered the full weekend (all days during the weekend are working or free days) and stand-alone shifts (no single shifts between free days) as the most important objectives in the problem. In (Parr and Thompson, 2007) the day off after a long shift (equivalent to night or watch shift in our case) and the cover constraint are considered as the most important objectives in the problem. In our case the balanced workload and non-consecutive night or watch shifts are considered as the most important objectives in the problem.

Table 2. Hard constraints.

	Hard constraint
H1	Shift coverage requirements must be fulfilled.
H2	Only one shift can be assigned for the nurse of the same position on the
	same day.
H3	At most two night shifts in seven-day period.
H4	At most forty hours must be worked in one week.
H5	At least twenty four hours rest must be after night shift.
H6	Morning, day, night shifts must be assigned only on working days.
H7	Watch shift must be assigned only at weekends or bank holidays.
H8	Schedules of nurses working on several positions must not overlap.
H9	Must be no empty spaces between the assignments of nurses working on
	several positions on the same day.

The hard constraints considered in the problem that we analysed are listed in the Table 2.

The government regulations do not allow working more than two night shifts in a 7day period, or more than 40 hours per week; and there must be at least 24 hours of rest time after a night shift. A shift is treated as night shift if it covers at least 3 hours of night time (the government regulations state that night time is between 22:00 and 06:00). Watch shift is also considered as night shift, the difference being that watch shift is assigned during weekends and bank holidays, while night shift is only on workdays. Remaining constraints are based on hospital regulations.

Notations used in this article to formulate the problem are as follows.

 $a_d^s$  is the amount of shifts  $s \in S$  required a day d, d = 1, 2, ..., D.

*B* is the set of nurses having several qualifications (skills).

 $C_i$  is the schedule of a nurse j.

 $dw_i$  is the demand workload in hours for a nurse j.

 $d_i^{off}$  is the requested day off by a nurse j.

 $d_s$  is the duration of shift  $s \in S$ .

 $d_{s,i}$  is the duration of a shift  $s \in L_i$  at index *i*.

D is the number of days in scheduling period (it may vary from 28 to 31).

 $et_{s,d}$  is the shift s end time on day d.

*Fd* is the indices of free days in scheduling period.

 $L_j$  is the set of current month assignments with previous month included last days assignments of a nurse j. The number of assignments taken from previous month so that  $|L_j| = 35$ .

N is the set of nurses.

 $rn_j$  is the requested number of night shifts to be assigned during the scheduling period for a nurse j.

 $rd_{j}$  is the requested number of watch shift to be assigned during the scheduling period for nurse j.

 $r_j^d$  is the preferred shift by a nurse j on weekday d, d = 1, 2, 3, 4, 5 (1 – Monday, 2 – Tuesday, ...).

S is the set of shifts, S = 1, 2, 3, 4 (1 - morning, 2 - day, 3 - night, 4 - watch).

 $st_{s,d}$  is the start time of a shift s on day d.

 $u_j$  is the unwanted shift by a nurse j.

W is the indices of working days in scheduling period.

 $x_{d,j}^{s}$  is the decision variable, if a nurse j is working on day d in shift  $s \in S$  its value is equal to 1, 0 otherwise.

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 $x_{d,j}$  is the decision variable, if a nurse j is working on day d its value is equal to 1, 0 otherwise.

 $\lambda_{d,j}^{s}$  is the decision variable, if a nurse j has assigned shift  $s \in S$  in set  $L_{j}$  at index d (d = 1,...35) its value is equal to 1, 0 otherwise.

Mathematical formulations of constraints H1 - H9 are expressed in formulas (1) – (8).

(H1) 
$$\sum_{d=1}^{D} \sum_{j \in N} x_{d,j}^{s} = a_{d}^{s}, s \in S.$$
(1)

(H2) 
$$\sum_{\substack{s \in S \\ 7 \ k+1}} x_{d,j}^{s} \le 1, d = 1, \dots, D, j \in N.$$
(2)

(H3) 
$$\sum_{d=7k+1}^{1} \lambda_{d,j}^s \leq 2,$$
 (3)

 $k \in [0,1,2,3,4], j \in N, s \in [3,4]$ .

7 k+1

(H4) 
$$\sum_{i=7k+1}^{k} d_{s,i} \le 40,$$
 (4)

 $s \in L_i, k \in [0, 1, 2, 3, 4], j \in N.$ 

(H5) 
$$\begin{aligned} st_{s,d+1} &\geq et_{s,d} \\ d &= 1, \dots, 34, s \in L_j, j \in N. \end{aligned}$$
(5)

(H6) 
$$\sum_{d \in Fd} \sum_{j \in N} x_{d,j}^s = 0, s \in [1,2,3].$$
(6)

(H7) 
$$\sum_{d \in W} \sum_{j \in N} x_{d,j}^{s} = 0, s \in 4 .$$
(7)

(H8, H9) 
$$\begin{array}{c} s_{s_{1,d}} = e_{s_{2,d}}, \\ d = 1, \dots, D, j \in B, s_{1}, s_{2} \in C_{j}. \end{array}$$
(8)

The objectives considered in our nurse scheduling problem are the following. **Objective 1**. The balance of workload of nurses.

This objective tries to eliminate the need to pay extra money to nurses for extra work while there are nurses who haven't worked their time defined in their working contracts.

The objective function  $F^{1}$  of this objective can be formulated as:

$$F^{1} = \left| \sum_{s \in C_{j}} d_{s} - dw_{j} \right|, j \in N.$$
(9)

Objective 2. The minimization of amount of successive night shifts.

Despite it does not violate any laws and regulations, schedules with consecutive night or watch shifts are considered as poor schedules in our problem, because of possible burnouts and decreased morale of nurses.

The objective function  $F^2$  of this can be formulated as the following:

$$F^{2} = x_{j,d}^{s} + x_{j,d+2}^{s},$$

$$d = 1, \dots, D - 2, j \in N, s \in 3, 4.$$
(10)

Objective 3. The uniform distribution of night shifts for nurses.

The objective is to assign night shifts so that neither nurse works on several night shifts (unless the nurse requests it) while there are nurses with no assigned night shifts at all. To measure equality in night shifts assignments, we use the term *night shift tolerance*, defining the number of night shifts over-assigned. If its value is equal to 0, it means that there are no more than requested night shifts assigned to the nurse schedule; while if greater than 0, it means that there are more than requested night shifts assigned to the nurse schedule. The objective is to minimise the difference between nurses having the maximum value of night shift tolerance and nurses having the minimum value of night shift tolerance.

The objective function  $F^3$  of this objective can be formulated as:

$$F^{3} = \left| \max\left(\sum_{d=1}^{D} x_{j,d}^{s} - m_{j}\right) - \min\left(\sum_{d=1}^{D} x_{j,d}^{s} - m_{j}\right) \right|,$$
(11)  
$$j \in N, s = 3.$$

**Objective 4.** The uniform distribution of watch shifts for nurses.

The objective is similar to the previous one: to assign watch shifts so that neither nurse works on several watch shifts (unless the nurse requests it) while there are nurses with no watch shifts assigned at all. To measure equality in watch shifts assignment we use term *watch shift tolerance*, defining the number of watch shifts over-assigned. If its value is equal to 0 it means that there are no more than requested watch shifts assigned to the nurse schedule; while if greater than 0 it means that there are more than requested watch shifts assigned to the nurse schedule. The objective is to minimise the difference between nurses having the maximum value of watch shift tolerance and nurses having the minimum value of watch shift tolerance.

The objective function  $F^4$  of this objective can be formulated as:

$$F^{4} = \left| \max\left(\sum_{d=1}^{D} x_{j,d}^{s} - m_{j}\right) - \min\left(\sum_{d=1}^{D} x_{j,d}^{s} - m_{j}\right) \right|,$$
(12)  
$$i \in N, s \in [4],$$

**Objective 5.** The minimization of the violations to nurses' preferences such as requested shifts on/off each weekday and day off.

Satisfaction of various preferences of nurses such as requested days off, unwanted shifts and shifts preferred to work on separate weekdays are considered in this objective.

The objective function  $F^5$  of this objective can be formulated as:

$$F^{5} = \sum_{d \in W_{d}} x_{j,d}^{s1} + \sum_{d=1}^{D} x_{j,d}^{s2}, \ j \in N, s1 \neq r_{j}^{d}, s2 = u_{j}.$$
(13)

General objective is to minimize all the objective functions.

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$$F \to \min \ F^1, \dots, F^5 \ . \tag{14}$$

In multi-objective optimization, there does not typically exist one feasible solution that minimises all objective functions. Solutions are called Pareto optimal solutions that cannot be improved by any of the objectives without worsening others. This concept is called Pareto optimality (Burke et al., 2012; Marler and Arora, 2004; Pardalos et al., 2012).

Decision vector  $x^* \in X$  is Pareto optimal if there does not exist another  $x \in X$  such that  $f_i \ x \le f_i \ x^*$  for all *i* and  $f_j \ x < f_j \ x^*$  for at least one index *j* (Nikulin et al., 2012).

All solutions that satisfy the Pareto optimality condition are considered to be optimal solutions to the problem. From a set of solutions, the planner (person responsible for scheduling nurses) can pick one solution that satisfies best the main objectives.

### 3. Description of the Most Relevant Methods

As mentioned above, the selection of the most favourable algorithm for a real world problem is problematic. In our previous research (Liogys, 2012; Liogys and Zilinskas, 2012) we have shown, that for the data relevant to a large Lithuanian hospital, the following methods are most suitable to the single objective optimization of nurse rosters: shift sequence method and shift sequence based method with variable neighbour search enhancement, and simulated annealing. Therefore the multi-objective versions of these methods also seem much promising. In the present paper we compare the multi-objective version of the simulated annealing algorithm (SMOSA) proposed in (Suman and Kumar, 2006) with two newly proposed multi-objective algorithms based on shift sequences.

The algorithm of simulated annealing (SMOSA) for multi-objective nurse scheduling problem is proposed in (Suman and Kumar, 2006) which is the following:

- 1. Randomly generate initial solution X.
- 2. Evaluate all objective functions and put to Pareto set of solutions
- 3. Generate new solution Y in neighbourhood.
- 4. If solution Y is feasible (no violations to hard constraints) go to step 5, if Y is not feasible generate another solution Y based on X.
- 5. Compare the generated solution vector with all the solutions in Pareto set and update Pareto set if needed.
- 6. If Y is archived, i.e. accepted as Pareto optimal, accept it and set X = Y.
- 7. If Y is not archived, accept with probability P.
- 8. If Y is not archived and not accepted keep X as background for neighbour solutions.
- 9. Reduce temperature using selected cooling function.
- 10. Repeat 3-9, until temperature reaches 0 or predefined number of iterations is reached.

The generalization of the shift sequence based method, and the enhanced shift sequence based methods are similar to that above. The multi-objective shift sequence based algorithm (SSA) problem can be described as follows:

- 1. Generate shift sequences.
- 2. Evaluate shift each sequence.

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- 3. Order shift sequences of a nurse in non-decreasing order by its penalty.
- 4. Remove, if needed, from set of shift sequences the undesired ones.
- 5. Set initial and final counter values.
- 6. Randomly order the set of nurses.
- 7. Assign shift sequence, if it does not violate constraint, for each nurse from ordered set. If sequence violates any of hard constraints select the next one, and so on. Between sequences assign random number of free days.
- 8. After creating the schedule for a nurse, swap any pair of shifts between nurses in partial roster, so that not to worsen any of nurses schedule.
- 9. If some of hard constraints are violated using greedy local search (GLS) repair the violations.
- 10. Compare the generated solution vector with all the solutions in Pareto set and update Pareto set if needed.
- 11. Archive Y if Y is non-dominated by solutions in Pareto set.
- 12. Sort the nurses by their schedule penalty in non-increasing order.
- 13. Increase counter value and if final value is reached finish procedure and produce the solution, if not repeat step 7 12.

The shift sequence based method with variable neighbour search enhancement (SSSA) (Liogys and Zilinskas, 2012) is adapted to multi-objective optimization in the following way.

- 1. Generate shift sequences.
- 2. Evaluate each shift sequence.
- 3. Order shift sequences of a nurse in non-decreasing order by its penalty.
- 4. Construct initial roster
- 5. Set maximal iteration counter value
- 6. Generate new solution Y in neighbourhood X.
- 7. If Y is not feasible repair using GLS.
- 8. Compare the generated solution vector with all the solutions in Pareto set and update Pareto set if needed.
- 9. If Y is archived accept and set X = Y.
- 10. If Y is not archived, accept with probability P.
- 11. If Y is not archived keep X as background for neighbour solutions.
- 12. Increase iteration counter.
- 13. If counter value does not reach maximum allowed value repeat (6-13).

# 4. Experimental Investigation of the Efficiency of the Selected Algorithms

The schedules were built for 50 nurses under conditions described in Section 2. Our goal is to investigate the most effective method to solve our problem.

Computers with Intel Core i3-2120 3.30 GHz processors have been used for the experimentation. Each algorithm has been run repeatedly 10 times using the same starting solution. The computation time limit equal to 1 hour was used as a termination condition.

The size of the problem we are tackling is defined by 9180 constraints and 7790 variables.

Since multi-objective approach usually produces a set of the solutions called Pareto optimal solutions, the average amount of Pareto optimal solutions in our case found by SA method is 11 solutions, by SSA method -156, and by SSSA method -10.

To compare the results found by algorithms we use hypervolume indicator which measures the volume of the dominated portion of the objective space delimited by reference point (anti-optimal point), usually worst solution in the Pareto set. The higher hypervolume indicator value is the better set of solutions is said to be. For example, considering two Pareto sets A and B, the hypervolume indicator values A higher than B if the Pareto set A dominates the Pareto set B (Bringmann and Friedrich, 2010).

To calculate the hypervolume indicator value we use the realization of recursive, dimension-sweep algorithm (Fonseca et al., 2006) created by the authors of the algorithm that can be found at (WEB, 2010).

To measure hypervolumes of objective spaces we use (120, 1, 4, 4, 700) as a reference point, meaning that we are considering only solutions that have not more than 120 minutes of workload misbalance; not more than 1 two consecutive night shifts; not more than 4 violations to nurses requests not to assign more than a certain amount of night shifts; not more than 4 violations to nurses requests not to assign more than a certain amount of ertain amount of watch shifts and, finally, not more than 700 violations to other nurses requests, such as days off, preferred and unwanted shifts, in the roster. The results are listed in Table 3.

Method	Average Hypervolume Indicator Value	
SSA	25997	
SSSA	51444	
SA	36933	

 Table 3. Hypervolume indicator values.

Enhanced shift sequence based method (SSSA) finds solutions that dominate both solutions sets found by SA and SS methods (Table 3).

Looking at the relations between two main objectives (to balance the workload and minimize number of consecutive night shifts) in randomly picked solutions sets we can see that SS method finds wider set of solutions than other two methods (Figure 1) (point in the square is marked as the best trade-off, considering only those two objectives).

Complete solutions having best workload balance and minimal number of consecutive night shifts are the following:

- SA method: (96, 0, 2, 0, 660).
- SSA method: (96, 0, 3, 0, 666), (96, 0, 5, 2, 651).
- SSSA method: (96, 0, 2, 0, 657), (96, 0, 3, 2, 645).

Analysing data from all tests we can state that, if to eliminate consecutive night shifts, violations to preferred maximum number of night and watch shifts from the roster would be the main objective, the most suitable method would be SSSA, because it provides solutions with less workload misbalance, varying from 5 to 10 hours, while SA method – from 10 to 24 hours. SS method does not find any solutions that meet these criteria.



Figure 1. Relationship between workload balance and number of consecutive night shifts

#### 5. Conclusions

The performance of several methods for the nurse rostering problem is investigated experimentally, namely the following methods have been considered: the shift sequence method, the simulated annealing method, and the enhanced shift sequence method.

Test results show that the proposed shift sequence based method with variable neighbour search enhancement outperforms both the simulated annealing and the shift sequence based method.

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